10

VISCOSITY AND BERNOULLI'S THEOREM

10.1 INTRODUCTION

In the previous lesson you have studied about the hydrostatic pressure and surface tension of liquids, which deals with the stationary liquids. Now you shall study about some of the chracteristic properties of liquids in motion i.e *hydrodynamics*.

You would have noticed that when you tilt a bucket filled with any liquid on to a horizontal ground, the liquid spreads on the ground. It does not flow continuously, but comes to rest after spreading to small distance. In this lesson you will learn about such peculiar property of liquids.

You have also learned that the liquids are incompressible or they show nearly zero compressibility. That is why, they do not show elastic behaviour. They can not be squeezed into a smaller volume like elastic solids. However, you know that water flows through cylindrical tubes of different cross-sections. The velocity of flow changes when it has to flow from a wider tube to a narrower tube. Many of you must have experienced that when you press the mouth of a soft-plastic or rubber water-pipe, while watering the dawn or plants, the stream of water falls at a larger distance away from the pipe held in your hand at a certain height above the ground. Do you know why? This can be explained on Bernoulli's principle. There are some other interesting activities which you must have seen or performed yourselves whose explanation can be undestood by the study of this lesson.

10.2 OBJECTIVES

After studying this lesson, you should be able to:

- *differentiate between streamline and turbulant motion;*
- *define critical velocity of flow of liquid and Reynold's number;*
- *explain different daily life phenomena based on viscosity effects of liquids;*
- *state Bernoulli's principle and explain different daily life phenomena based on Bernoulli's principles.*

10.3 VISCOSITY

All matter, when in liquid state develops one more characteristic property, (besides surface tension about which you have studied in the earlier lesson) called ***viscosity*** of liquids.

If you drop a stone from the top of a high building, its velocity continuously increases till it reaches the ground due to the action of acceleration due to gravity; but if you drop the same stone, in deep water, the velocity of the falling stone initially increases but after certain time, before reaching the ground or bottom of the container, attains a constant velocity showing that the effect of acceleration due to gravity has been made ineffective. How it happnes?

The liquid when in slow motion moves in layers but when in fast motion, flows turbulantly. Why? Why does the glycerine flow slowly while water flows quickly down the same inclined plane?

All these are due to the viscous nature of liquids.

10.3.1 Stream-line motion

The path followed by an element of a moving liquid is called a line of flow. *If every element passing through a given point of the path follows the same line of flow as that of preceeding elements, the flow is said to be* ***streamline.***

A stream line is defined as the curve whose tangent at any point gives the direction of the liquid velocity at that point. In steady flow, the streamlines coincide with the line of flow.

The two streamlines do not intersect each other (in a stream line flow), because two tangents, can be drawn at the point of intersection giving two directions of velocities which is not possible.

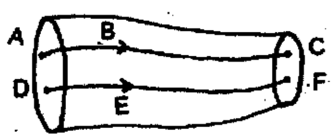

Fig. 10.1: *Stream-line flow*

When the velocity of flow υ is less than the critical velocity υ_c for a given liquid flowing through tube of given diameter, the motion is streamlined. In such a case we can imagine the entire thickness of the stream of the liquid made up of large number of plane layers (laminas) one sliding past the other i.e. one flowing over the other. Such a flow is called **laminar flow.**

If the velocity of flow exceeds the critical velocity υ_c, the mixing of streamlines takes place and the flow path becomes zig-zag, the streamlines cutting each other and the motion is said to be ***turbulant.***

10.3.2 Viscosity as liquid friction

The stream line motion results in a laminar flow when the liquid is flowing over a horizontal surface. The top most layer is moving (sliding over the lower layers of the liquid) with maximum velocity; the layer just below the top most layer is touching the upper layer as well as pressed by the weight of the upper layer. Hence moves with a slightly lower steady velocity. This is said to be due to the liquid friction between the two successive layers being operative all along the surfaces of contact.

Since the layers are in contact with each other, the upper layer moving with higher velocity tends to accelerate the motion of the lower layer and the tower layer moving with smaller velocity tends to retard the motion of the upper layer i.e. the layers, in laminar flow, tend to destroy the relative motion between them.

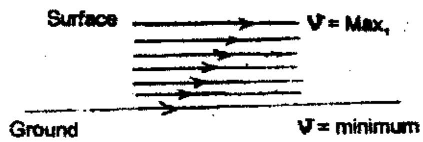

Fig. 10.2 : *Laminar flow of a Liquid*

This tendency of the liquid by virtue of which it tends to destroy the relative motion between its adjascent laminar layers is called viscosity. Due to this nature, a liquid friction force or tangential backward force acts between the two layers and is called the ***viscous force.***

It is thus clear that due to this viscous force the velocity of the upper layer gets decreased to the value of the lower layer in contact with it. Similarly, the velocity of the successive layers goes on decreasing due to this tangential backward viscous force till all the layers keep moving with the velocity of the lowest layer. But the lowest layer is touching the ground and can be assumed to be staionary due to large friction between solid ground and liquid. It, therefore, results that the liquid in laminar flow should stop flowing after covering some distance.

To maintain the laminar flow, therefore, some external force should be applied to counter the effect of viscous force. This is usually done by creating a pressure difference between the two points between which the laminar fow is to be maintained. Fig (10.2) shows the laminar flow of a liquid on a horizontal surface.

Suppose, the velocity of a layer of a height x, from the ground is υ and that of the next upper layer at a distance $(x+\Delta x)$ is $(\upsilon+\Delta\upsilon)$, then it is known as *velocity gradient* and it is given as follows.

The rate of increase of velocity with the height

(or distance from the ground)

$$= \frac{(\upsilon+\Delta\upsilon)-(\upsilon)}{(x+\Delta x)-(x)} = \frac{\Delta\upsilon}{\Delta x}$$

If the limit $\Delta x \to 0$, $\Delta\upsilon$ will also be small and

we can write $\frac{\Delta\upsilon}{\Delta x} = \frac{d\upsilon}{dx}$ = differential of 'υ' w.r.t. 'x'.

= velocity gradient.

According to *Poiseville*, in a streamlined or laminar flow of viscous fluids, the magnitude of the tangential backward (opposing the flow) viscous force is proportional to

(i) the area of contact (A) of the liquid surfaces; and

(ii) the velocity gradient $\left(\frac{d\upsilon}{dx}\right)$ *of the region in which the layers are moving*

i.e. $F \alpha A$

and $\alpha \frac{d\upsilon}{dx}$

i.e. $F \alpha A \frac{d\upsilon}{dx}$

or $F = \eta A \frac{d\upsilon}{dx}$ ... (10.1)

where η = constant of proportionality and is called the ***coefficient of viscosity***, which is measure of the viscous nature of the fluids.

It is measured in Poise in C.G.S. system where 1 Poise = 1 g cm^{-1} s^{-1}.

However in SI system, its practical unit is $Nm^{-2}s$.

Its dimensional formula is $\left[ML^{-1}T^{-1}\right]$ and hence its theroretical unit is $\left(kg\,m^{-1}s^{-1}\right)$

Also $1Nm^{-2}s = 1\ kg\ m^{-1}\ s^{-1} = 10$ Poise.

$$h = \frac{F}{A.\left(\frac{dv}{dx}\right)}$$

Since,

$$\text{If } A = 1m^2;\ \frac{du}{dx} = 1s^{-1},\ \text{then } h = F$$

Hence, ***coefficient of viscosity*** *of any fluid is defined as equal to the tangential backward viscous force acting on a unit area of a laminar layer moving in a region of unit velocity gradient.*

10.3.2 Critical Velocity and Reynond's Number

As you have seen earlier, when the velocity of flow is small i.e. less than a certain value, called *critical velocity*, the flow remains *streamlined;* when the velocity of flow exceeds the critical velocity, the flow becomes *turbulent* i.e they do not remain well defined streamlined paths.

The value of critical velocity of any liquid depends upon the

i) viscous nature of the liquid (η)

ii) the diameter of the tube *(d)* through which the liquid is flowing and

iii) also on the density of the liquid *(ρ)*

If η = coefficient of viscosity of the liquid, which is measure of the viscous nature (or liquid friction) of the liquid, then it is found experimentally that

critical velocity (υ_c)

$$\upsilon_c \alpha \eta;\ \upsilon_c \alpha \frac{1}{\rho};\ \upsilon_c \alpha \frac{1}{d}$$

$$\upsilon_c \alpha \frac{\eta}{\rho d}$$

Or $$\upsilon_c = R\frac{\eta}{\rho d} \quad \text{... (10.2)}$$

Where, R = constant of propertionality and is called ***Reynold's Number***. It has no dimensions. It has been observed that for all liquids, the value of Reynold's number corresponding to υ_c is nearly the same i.e. $R = 2000$.

Thus, for water $\upsilon_c = \dfrac{2000 \times 0.01}{1 \times 1} = 20\,cm\,s^{-1}$

[using, $\rho = 1\,gm/cm,\ d = 1cm,\ \eta = .01\,poise$]

Thus for water, the flow of velocity greater than $0.2ms^{-1}$ will become turbulent while flowing through a tube of diameter 0.01m. i.e. if the water flows at a speed of 0.25 cms^{-1}, its motion through this tube will be turbulant.

If, however, another liquid of density 1.2 gm/cc and coefficient of viscosity 0.02 poise flows through the same tube of diameter 1 cm, the critical velocity

$$\text{is } = \upsilon_c = \frac{2000 \times 0.02}{1.2 \times 1} = 33.3cms^{-1}$$

Therefore, if this liquids also flows with a speed of $25cms^{-1}$, its motion will remain streamlined (because $\upsilon = 25\ cms^{-1} < \upsilon_c = 33.3\ cms^{-1}$).

If $\bar{\upsilon}$ = average speed of flow of a liquid on any surface then Reynold number R. can be expresed as, from equation

$$R = \frac{\bar{\upsilon} d\rho}{\eta} \qquad \text{... (10.3)}$$

Experiments show that flow is laminar if R has a value less than **2000** but is turbulant when R exceeds this value.

Example 10.1 : *The average speed of blood in the artery (d = 2.0 cm) during the resting part of heart's cycle is about 30 cms^{-1}. Is the flow laminar or turbulant? Density of blood 1.05 gm cm^{-3}; $\eta = 4.0 \times 10^{-2}$ poise.*

Solution : Let us calculate the Reynold's number to decide about this.

$$\text{Since } R = \frac{\bar{\upsilon}\, d\rho}{\eta}$$

$$\therefore\ R = \frac{30 \times 2 \times 1.05}{4.0 \times 10^{-2}} = 1575$$

Since 1575 << 2000; the flow is laminar, however, it is close to turbulant flow.

10.3.3 Stoke's law

Sir George Stokes gave an expirical law for the magnitude of the tangential backward viscous force (F) acting on a freely falling smooth spherical body of radius (r) in a highly viscous liquid of coefficient of viscosity (η) moving with velocity (υ) which is known as Stoke's law.

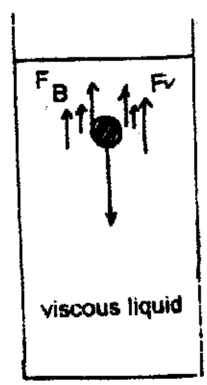

Fig. 10.3

According to Stoke's law

$F \alpha \eta;\ F \alpha r;\ F \alpha \upsilon$

i.e. $F \alpha \eta r\upsilon$

or $F = K' \eta r \upsilon$; where K' = contstant of proportionality. Experimentally, it has been found that $k' = 6\pi$, hence

Stoke's law is given by $\boxed{F = 6\pi\eta\, r\upsilon}$ ... *(10.4)*

Let us consider a small spherical heavy ball of radius r to fall in the highly viscous liquid like glycerine, contained in a long jar as shown in fig. (10.3) to fall freely.

The moment the sphere enters the liquid, it starts experiencing a buoyant force $F_B = \left(\frac{4}{3}\pi r^3\right)d_l g$

Where d_l is the viscosity of the liquid.

where d_l = density of the viscous liquid

If υ = velocity of the liquid at any instant of time, then it also experiences a tangential backward viscous force F_v, given by

$$F_v = 6\,\pi\eta r\,\upsilon$$

The weight = $W = \left(\frac{4}{3}\pi r^3 . d_s\right)g$ acts downwards $= mg.$; where d_s = density of solid ball-material.

Initially $W > F_V + F_B$

and the spherical ball keeps on falling with acceleration

$$a = \frac{W - (F_V + F_B)}{m}$$

and hence continuously gain in velocity will result continuous increase in the value of F_V also.

A situation may arise, when F_V becomes so large that

$$W = F_V + F_B$$

and then $a = 0$; and the spherical ball then attains a constant velocity υ_t, called, the ***terminal velocity.*** Thereafter, the ball falls with constant velocity υ_t due to inertia of motion.

In such a case,

$$\frac{4}{3}\pi r^3 . d_s g = 6\pi\eta r\,\upsilon_t + \frac{4}{3}\pi r^3\, d_{lg}$$

Thus, gives $\boxed{\upsilon_t = \frac{2}{9}\frac{r^2 g(d_s - d_l)}{\eta}}$ ... (10.5)

Table 10.1 gives the values of coefficient of viscosities of some common viscous liquids :

TABLE 10.1 : η for some liquids

S.No.	Liquids	Temperature °C	Coefficient of viscosity $kg\ m^{-1}s^{-1}$
1.	water	0	1.792×10^{-3}
		20	1.005×10^{-3}
		40	0.636×10^{-3}
		60	0.469×10^{-3}
		80	0.357×10^{-3}
		100	0.284×10^{-3}
2.	Honey	20	2×10^{-2}
3.	Blood	37	1.5×10^{-4}
4.	Glycerine	20	149×10^{-2}
5.	Turpentine oil	20	1.49×10^{-3}
6.	Benzene	20	6.5×10^{-4}
7.	Light machine oil	16	1.13×10^{-2}
8.	Tin oil	20	4.0×10^{-3}
9.	Olive oil	20	84×10^{-3}
10.	Air	0	1.71×10^{-5}

Example 10.2: *Determine the radius of the drop of rain water falling through air, with terminal velocity 0.12 ms^{-1}. Given η for air = 1.8×10^{-5} $kgm^{-1}s^{-1}$ and density of air = 1.21 kgm^{-3}.*

Solution:

Since, $$\upsilon_t = \frac{2r^2 g(d_s - d_l)}{9\eta}$$

$$r = \sqrt{\frac{9\eta\upsilon_t}{2g(d_s - d_l)}} = \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 0.12}{2 \times 9.8 \times (1000 - 1.21)}}\ m$$

i.e., radius of the drop = $r = 10^{-5}$ m = 0.01 mm

Now, let us check how much you have learnt. Try to solve the following questions.

INTEXT QUESTION 10.1

1. *Differentiate between streamline flow and turbulent flows.*

   ....

2. *On what physical quantities does critical velocity depend for a viscous liquid?*

   ....

3. *Choose the correct answers*

(i) *The value of η of any liquid depends upon*

(a) nature of the liquid; (b) radius of the ball;

(c) *density of the liquid;* (d) *on all the above physical quantities*

(ii) *Glycerine and water are pushed on the horizontal ground with the same initial jerk by tilting their containers*

(a) *The water will move through larger distance before coming to rest*

(b) *Glycerine will move through longer distance before coming to rest*

(c) *Both will move through the same distance before coming to rest. however, glycerine will take longer time than water.*

4. *Which of the following will be the better lubricant to be used in the rotating pents of a machine.*

(a) *water;* (b) *terpentine oil;* (c) *air*

5. *The terminal velocity of copper sphere of radius 4.00 mm in falling through a tank of oil at 50°C is 0.26 ms^{-1}. Compute the viscosity of the oil at 50°C. Density of oil = 1.5×10^3 kg m^{-3}; density of copper = 8.9×10^3 kg m^{-3} and g = 9.8 ms^{-1}*

....

Pressure energy

Earlier you have studied about the kinetic energy and potential energy of a body, but in case of liquids there is a pressure energy also. If a liquid element of mass = m, density = d is moving under a pressure difference = p, then it has pressure energy = pressure difference × volume

$$= p \times \left(\frac{m}{d}\right) \text{ Joule}$$

or pressure energy per unit mass $= \frac{p}{d}$ Joule

10.4 BERNOULLI'S PRINCIPLE

Have you ever thought, how air circulates in a dog's burrow; why smoke comes out quickly out of a chimney; or why car's convertible top bulges upward at high speed? You must have definitely experienced the bulging upwards of your umbrella on a stormy-rainy day. All these can be understood on a principle worked out by Daniel Bernoulli (1700-1782).

Bernoulli's principle states that '***where the velocity of fluid is high, the pressure is low and where the velocity of fluid is low, the pressure is high***'.

10.4.1 Bernoulli's Equation

Bernoulli developed an equation that expresses this principle quantitatively. Three important assumptions are to be made to develop this equation.

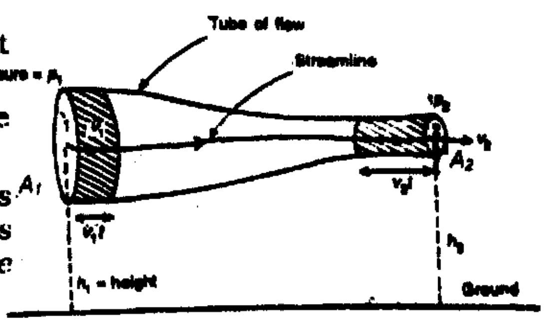

Fig. 10.4

1. *The fluid is incompressible i.e., its density does not change when it passes from a wide bore-tube to a narrow-bore tube or vice versa.*

2. *The fluid is non-viscous or the effect of*

viscosity is not to be taken into account in this derivation.

3. *The motion of the fluid is stream-lined.*

We consider a tube of flow. Let AB be a streamline. Suppose at point **A** The pressure $=p_1$, area of cross-section $= A_1$, velocity of flow $= v_1$, height above the ground $= h_1$, and **at B** The pressure $= p_2$, area of cross-section $= A_2$, velocity of flow $= v_2$, height above the ground $= h_2$.

When the liquid at A moves through a distance Δl_1, it forces the liquid at B to move through a distance Δl_2 (which is different from Δl_1 due to different cross-sectional areas at A & B).

The fluid to the left of point A exerts a pressure p_1 on the fluid pushing it in the right direction.

The amount of work done in displacing through a small distance Δl_1 is

$$= W_1 = (P_1 \times A_1) \times (Al_1).$$

At point B, the liquid does work on the liquid to the right of it by moving through a distance Δl_2 given by $= W_2 = (P_2 \times A_2) \times (\Delta l_2)$.

$\therefore$ Net work done on the liquid $= W_1 - W_2 = (P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2)$

Since the liquid is not moving horizontally, work is done against gravity also which is stored as the extra potential energy of the liquid at B

Now Gain in extra potential energy = (P.E. at B – P.E. at A)

$= mg(h_2 - h_1)$; where m = mass of the liquid transferred from A to B.

The total work done on the liquid

$$= (W_1 - W_2) + mg(h_2 - h_1)$$

According to the law of conservation of energy, the net work done on the liquid is equal to change in its total energy *i.e.*, sum of kinetic energy and potential energy. Therefore,

$$\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + mg(h_2 - h_1) = W_1 - W_2$$

$$= p_1 A_1 \Delta l_1 - p_2 A_2 \Delta l_2$$

but $A_1 \Delta l_1 = A_2 \Delta l_2 = \dfrac{m}{d}$ = Volume of the liquid flown

$$\therefore \quad \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 + mgh_2 - mgh_1 = p_1 \frac{m}{d} - p_2 \frac{m}{d}$$

$$\text{or} \quad \frac{mp_1}{d} + \frac{1}{2}mv_1^2 + mgh_1 = \frac{mp_2}{d} + \frac{1}{2}mv_2^2 + mgh_2$$

This is Bernoulli's equation. Since points A and B can be any two points along a tube of flow, Bernoulli's equation can be written as

$$\frac{mp}{d} + \frac{1}{2}mv^2 + mgh = constant \qquad ...(10.6)$$

which can now be states as —

The sum of pressure energy, kinetic energy and potential energy of a fluid remains constant in a stream line-motion.

10.4.2 Applications of Bernoulli's Theorem

Bernoulli's theorem finds many applications in our day to day lifes. Some commonly observed phenomena can also be explained on the Bernoulli's Theorem.

Flow Meter or Venturimeter

It is a device used to measure the rate of flow of liquids through pipes. The device is inserted in the flow pipe. Fig. 10.6

It essentially consists of manometer, with its two limbs connected to a tube having two different cross-sectional areas say A_1 and A_2 at its ends A and B respectively.

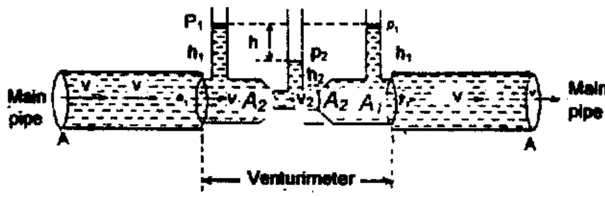

Fig. 10.5 *Venturimeter having three vertical manometric tubes is inserted in the main pipe line.*

Suppose the main pipe of flow is kept horizontal at a height H above the ground. Then applying Bernoulli's theorem for the steady flow of liquid through the venturimeter at its two points A and B, we can write

Total Energy at A = Total Energy at B

$(KE + PE + P_rE)_A = (KE + PE + P_rE)_B$

i.e., $$\frac{1}{2}mv_1^2 + mgH + \frac{mp_1}{d} = \frac{1}{2}mv_2^2 + mgH + \frac{mp_2}{d}$$

which gives $$(p_1 - p_2) = \frac{1}{2}d\left(v_2^2 - v_1^2\right) \quad ...(10.7)$$

Thus points of higher velocities are the points of lower pressure (because of the sum of P_rE and KE remain constant).

This is called ***Venturi's Principle.***

For steady flow through the venturimeter liquid volume entering per second at A = liquid volume leaving per second at B

i.e. $A_1v_1 = A_2v_2$ ...*(10.8)* [The liquid is assumed incompressible]

i.e., velocity is more at narrow ends and vice versa.

Using this relation (10.8) in relation (10.7), we conclude that pressure is lesser at the narrow ends, and get

$$p_1 - p_2 = \frac{1}{2}d\left[\frac{A_1^2 v_1^2}{A_2^2} - v_1^2\right]$$
$$= \frac{1}{2}dv_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]$$

This gives $$\boxed{v_1 = \sqrt{\frac{2(p_1 - p_2)}{d\left(\frac{A_1^2}{A_2^2} - 1\right)}}} \quad ...(10.9)$$

If h = level difference between the two limbs of the venturimeter then $p_1 - p_2 = hdg$

This gives
$$v_1 = \sqrt{\frac{2hg}{\left(\frac{A_1^2}{A_2^2} - 1\right)}}$$

or $\quad v_1 \propto \sqrt{h}$; since all other parameters are constant for a given venturimeter

$$v_1 = K\sqrt{h} \text{ ; where } K = \text{constant}$$

∴ Volume of liquid flowing per second

$$V = A_1 v_1 = A_1 \times K\sqrt{h}$$
$$V = K'\sqrt{h}; \quad when \quad K' = KA_1$$

The appratus can be directly caliberated to give A.

The venturi principle tells that since $A_1 v_1 = A_2 v_2 \rightarrow$ the velocity increases at narrower throat of the pipe and at the narrower throat the pressure is lesser *i.e., the pressure is lower near a narrow throat when the liquid flows steadily*. This principle has many applications in the design of many useful appliances like atomizer, spray gun, Bunsen-burner, carburettor, Aerofoil, etc.

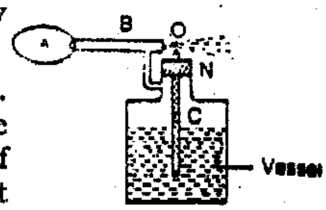

Fig. 10.7: *Atomizer*

(i) Atomizer : When the rubber bulb A is squeezed, air blows through the tube B and comes out of the narrow orifice with larger velocity creating a region of low pressure in its neighbourhood. The liquid (scent or paint) from the vessel is, therefore, sucked into the tube) to come out of the nozzle N. As the liquid reaches the nozzle N, the air stream from the tube B blows it into a fine-spray. Fig. 10.7.

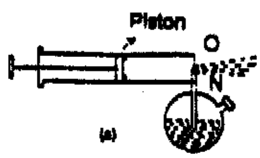

Fig.10.8: *Spraygun*

(ii) Spray gun : When the piston is moved in, it blows the air out of the narrow hole 'O' with large velocity creating a region of low pressure in its neighbourhood. The liquid (insecticide) is sucked through the narrow tube attached to the vessel end having its opening just below 'O'. The liquid on reaching the end gets sprayed by out blown air from the piston. Fig. 10.8.

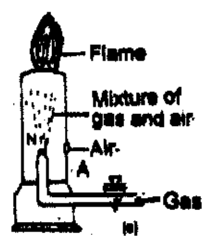

Fig. 10.9; *Bunsen Burner*

(iii) Bunsen burner ; When the gas emerges out of the nozzle N, its velocity being high, the pressure becomes low in its viscinity. The air, therefore, rushes in through the side hole A and gets mixed with the gas. The mixture then burns at the mouth when ignited, to give a hot blue flame Fig. 10.9.

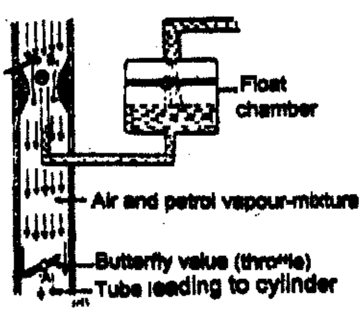

Fig.10.10:*Carburettor*

(iv) Carburettor : The carburettor shown in figure 10.10 is a device used in motor cars for supplying a proper mixture of air and petrol to the cylinder of the engine. The energy is supplied by the explosion of this mixture inside the cylinders of the engine. Petrol is contained in the float chamber. There is a decrease in the pressure on the side A due to motion of the piston. This causes the air from outside to be sucked in with large velocity. This causes a low pressure near the nozzle B (due to constriction, velocity of air sucked is more near B) and, therefore, the petrol comes out of the nozzle B which gets mixed with the incoming air. The mixture of vapourized petrol and air forming the fuel then enters the cylinder through the tube A.

[Sometimes when the nozzle B gets choked due to deposition of carbon or some impurities, it checks the flow of petrol and the engine not getting fuel stops working. The nozzle has, therefore, to be opened and cleaned.]

(v) Aerofoil : When a solid moves in air, streamlines are formed. The shape of the body of the aeroplane is designed specially as shown in the fig. 10.11. When the aeroplane runs on its runway, high velocity streamlines of air are formed by the passing air. Due to crowding of more streamlines on the upper side that becomes a region of more velocity and hence of comparatively low pressure region then below it. This pressure diefference gives the lift to the aeroplane.

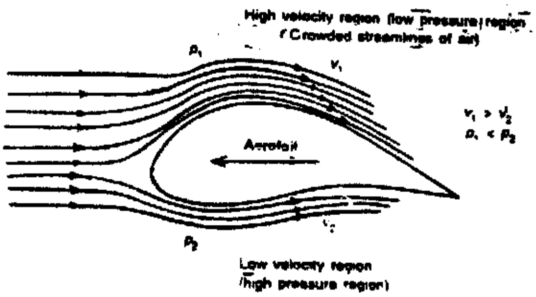

Fig. 10.11: *Crowding of streamlines on the upper side*

Based on this very principle *i.e.*, the regions of high velocities due to crowding of steamlines, are the regions of low pressure, following are interesting demonstrations.

(a) Attracted disc paradox: when air is blown through a narrow tube handle into the space two **cardboard sheets** [Fig. 10.12 (a)] placed one above the other and the upper disc is lifted with the handle, the lower disc is attracted to stick to the upper disc and is lifted with it. This is called attracted disc paradox.

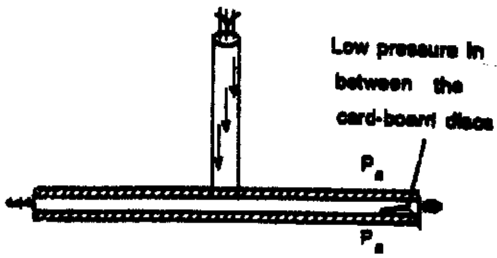

Fig.10.12: *Attracted disc paradox*

(b) Dancing of a ping pong ball on a jet of water: If a light hollow spherical ball (ping-pong ball or table tennis ball) is gently put on a vertical stream of water coming out of a vertically upward directed jet end of a tube, it keeps on dancing this way and that way without falling to the ground. Fig. 10.13. When the ball shifts to the left, then most of the jet streams pass by its right side thereby creating a region of high velocity and hence low pressure on its right side in comparison to that on the left side and the ball is again pushed back to the centre of the jet stream.

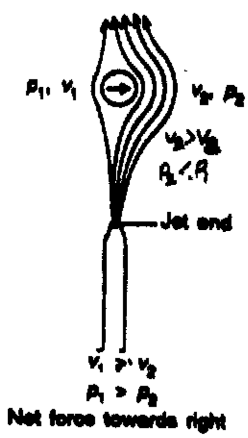

Fig.10.13: *Dancing Ping Pong ball*

(c) Water vacuum pump or Aspirator or Filter pump: Fig. 10.14 shows the filter pump used for producing moderately low pressures. The water from the tap is allowed to come out of the narrow jet end of the tube A. Due to small aperture of the nozzle, the velocity becomes high and hence a low pressure region is created around the nozzle N. The air is, therefore, sucked from the vessel to be evacuated through the tube B; gets mixed with the steam of water and goes out through the outlet. After a few minutes, the pressure of air in the vessel si decreased to about 1 cm of mercury by such a pump.

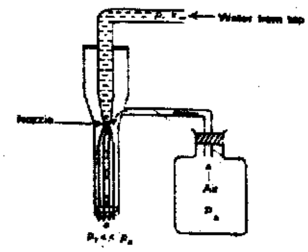

Fig.10.14: Filter Pump

Example 10.3: *Water flows out of a small hole in the wall of a large tank near its bottom. What is the speed of efflux of water when the height of water level in the tank is 2.5 m?*

Solution: Let B be the hole near the bottom. Imagine a tube of flow A to B for the water to flow from the surface point A to the hole B we can apply Bernoulli's theorem to the points A and B for the streamline flow of small mass m

Total energy at B = Total energy at A

$v_A = 0$ – (nearly zero)

$p_A = P$ = atmospheric pressure

h_A = height above the ground

$v_B = v = ?$

$p_B = P$ = atmospheric pressure

h_B = height of the hole above the ground

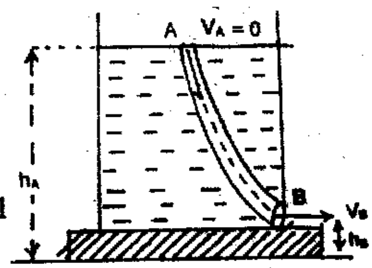

Fig.10.15

Let $h_A - h_B = H$ =height of the liquid level in the vessel

d = density of the liquid water

Applying Bernoulli's Principle

$$\frac{mp}{d} + \frac{1}{2}m \times (0)^2 + mgh_A = \frac{mp}{d} + \frac{1}{2}mv^2 + mgh_B$$

This gives $\quad \frac{1}{2}mv^2 = mg(h_A - h_B)$

or $\quad v = \sqrt{2g(h_A - h_B)}$ [This formula is the same as for a solid falling from a height h_B to h_A]

$$= \sqrt{2 \times 9.8 \times 2.5}$$

$$= 7\,\text{ms}^{-1} \quad \text{Ans.}$$

Take a pause and solve the following questions.

INTEXT QUESTIONS 10.2

1. *If the velocity at each point in space in steady state fluid flow is constant, how can a fluid particle accelerate without using any force i.e., gravitational or pressure difference?*

   ....

2. *When a car on a highway is passed by a larger truck, the car is sometimes pulled towards the truck. What does Bernoulli's theorem say about this?*

   ....

3. *What are the conditions necessary for the application of Bernoulli's theorem to the problems of flowing liuqids?*

   ....

4. *When you press the mouth of a water pipe used for watering the plants in the kitchen garden, the jet of water goes to longer distances. Why?*

....

5. *Calculate the velocity of wate of density 1000 kgm^{-3} coming out of a pin hole made at a height of 0.5 m above the bottom of a container, which is filled to a height of 0.9 m?*

....

10.5 WHAT YOU HAVE LEARNT

- The property by virtue of which the different layers of a liquid in its laminar flow tend to destroy the relative motion between them is called viscosity.
- The flow of liquids becomes turbulant when the velocity is greater than a certain value called critical velocity (v_c) which depends upon the nature of the liquid and the diameter of the tube *i.e.*, (η, ρ and d).
- The critical velocity $v_c = R\dfrac{\eta}{\rho d}$ where R = Reynold's number, which is nearly the same for all liquids and is $\approx$2000.
- Coefficient of viscosity of any liquid may be defined as the magnitude of tangential backward viscous force acting between two successive layers of unit area in contact with each other moving in a region of unit velocity gradient.
- Stoke's law states that tangential backward viscous force acting on a spherical mass of radius r falling with velocity v in a liquid of coefficient of viscosity η is given by $F_v = 6\pi\eta r v$
- The terminal velocity is achieved when weight is counterbalanced by the tangential backward viscous force and buoyant forces together *i.e.*, $W = F_v + F_B$

  The terminal velocity is then given by

$$v_t = \frac{2r^2 g(d_s - d_l)}{9\eta}$$

- The energy due to pressure (p) in a flowing liquid element of mass (m) and density (ρ) is given by

$$P.E = \frac{mp}{\rho}$$

- Bernoulli's theorem states that the total energy of an element of mass (m) of an incompressible liquid moving steadily remains constant thoughout the motion. Mathematically, Bernoulli's theorem as applied to any two points A & B of a tube of flow is

$$\frac{1}{2}mv_A^2 + mgh_A + \frac{mp_A}{\rho} = \frac{1}{2}mv_B^2 + mgh_B + \frac{mp_B}{\rho}$$

- Bernoulli's principle find many applications in our daily life like design of venturimeter, to measure the rate of flow of liquids in pipes, atomizer, spray gun, carburettor, filter pump, aerofil, etc.

10.6 TERMINAL QUESTION

1. Differentiate between Laminar flow and turbulant flow and hence define critical velocity.
2. Define viscosity and coefficient of viscosity. Derive the units and dimensional formula of coefficient of viscosity. Which is more viscous — water or glycerine. Why?
3. What is Reynold's number? What is its significance? Define critical velocity on the basis of Reynold's number.
4. State Bernoulli's principle. Explain its application in the design of the body of an aeroplane.

5. What is pressure energy? Derive Bernoulli's equation for flow of fluids.

6. Explain why :

   (i) A spinning tennis ball curves during the flight?

   (ii) A ping pong ball keeps on dancing on a jet of water without falling on to either side?

   (iii) The velocity of flow increases when the aperture of water pipe is decreased by squeezing its open mouth?

   (iv) A small spherical ball falling in a viscous fluid attains a constant velocity after some time?

   (v) If mercury is poured on a flat glass plate, it breaks into small spherical droplets?

7. Find out the terminal velocity of an air bubble with 0.8 mm in diameter which rises in a liquid of viscosity of 0.15 $kgm^{-1}s^{-1}$ and density 0.9 $gmcm^{-3}$. What will be the terminal velocity of the same bubble while rising in water? For water $\eta = 10^{-2}$ $kgm^{-1}s^{-1}$. [Ans.0.0209 cms^{-1}; 34.8 cms^{-1}]

8. A pipe line 0.2 m in diameter, flowing full of water has a constriction of diameter 0.1 m. If the velocity in the 0.2 m pipe-line is 2 ms^{-1}, find :

   (i) the velocity in the constriction, and

   (ii) the discharge rate in cubic meters per second.

9. (i) With what velocity in a steel ball 1 mm is radius falling in a tank of glycerine at an instant when its acceleration is one-half that of a freely falling body?

   (ii) What is the terminal velocity of the ball? The density of steel and of glycerine are 8.5 gm cm^{-3} and 1.32 gm cm^{-3} respectively; viscosity of glycerine is 8.3 Poise.

10. Water at 20°C flows with a speed of 50 cm s^{-1} through a pipe of diameter of 3 mm.

    (i) What is the Reynold's Number?

    (ii) What is the nature of flow?

    Given, viscosity of wate at 20°C as = 1.005×10^{-2} Poise; and

    Density of water at 20°C as = 1 gm cm^{-3}.

11. Modern aeroplane design calls for a lift of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the velocity of flow past the lower wing surface is 100 ms^{-1}, what is the required velocity over the upper surface to give a desired lift of 1000 Nm^{-2}? The density of air is 1.3 kg m^{-3}.

12. Water flows horizontally through a pipe of varying cross-section. If the pressure of water equals 5 cm of mercury at a point where the velocity of flow is 28 cms^{-1}, then what is the pressure at another point, where the velocity of flow is 70 cms^{-1}? [Tube density of water 1 gm cm^{-3}].

ANSWER TO THE TERMINAL QUESTIONS

7. 0.0209 cms^{-1}, 34.8cms^{-1}

8. (i) 8 ms^{-1}, (ii) 0.0628m^3s^{-1}

9. (i) 0.769 cms^{-1} (ii) 1.89 cms^{-1}

10. (i) 1500, (ii) Streamline flow

11. 107ms^{-1}

12. 4.845 cm^{-1} of Hg