14

ELECTRIC CHARGE AND ELECTRIC FIELD

14.1 INTRODUCTION

In our day to day life, we are dependent on electricity. An electric power failure demonstrates our dependence – lights in our homes/schools go out, fans and/or coolers stop working, radio. T.V. and computer cannot be operated, in village field cannot be irrigated as pumps fails, electric trains and elevators stop. This list is not complete and you can add many more things. Our dependence on electricity runs even deeper than our reliance on electrical machinery and home appliances/gadgetry. As you all know, electricity is inherent in all the atoms – of which our own body and environment is made of. Thus our immediate environment-behaviour is dominated by the behaviour of electric charges it contains. We know that matter consists of elementary charged particles: electrons and protons and neutrons. In the following lesson we will study about the frictional electricity, nature of electric charge and then about the electric field produced by them. We will also learn about electric forces and their effects. Broadly speaking all these come under the heading of electrostatics which is a branch of physics dealing with charges at rest.

14.2 OBJECTIVES

After studying this lesson, you should be able to :

- *explain frictional electricity and give the account of its historical development;*
- *state basic properties of the electric charge and explain the meaning of quantisation and conservation of charges;*
- *state Coulomb's law and write it in vector notation ;*
- *explain the principle of superposition;*
- *define electric field at a point due to an electric charge;*
- *draw electric field lines of a charge or two opposite charges;*
- *define electric dipole and its moment and derive expression for the electric field intensity due to an electric dipole;*
- *state Gauss-theorem and its significance and use it for determining electric field of a point charge, long wire, spherical shell, solid sphere, plane sheet of charge.*

14.3 FRICTIONAL ELECTRICITY

We know that when a glass rod is rubbed with a piece of silk cloth, the rod develops a property of attracting small pieces of paper, leaves etc. towards it. This rod is said to be ***charged or electrified.*** Similarly a comb is charged on passing through dry hair. An electric charge is developed on a sheet of paper moving through a printing press and so on. The charges at rest are produced due to friction between two insulating bodies which are rubbed against each other.

The knowledge of ***frictional electricity*** dates back to 600 B.C. We find, in writings of early Greek philosopher Thales that ***amber*** (the gum that long ago, oozed from softwood trees hardened into a semitransparent solid, ranging in colour from yellow to brown) acquires the property to attract the small pieces of light matter, when it is rubbed against the fur or wool. The *Greek* name of amber is "Electrum", which is origin of the words electric charge, electricity, electron etc. In 1600 A.D., William Gilbert after nearly 2000 years of Thales observation, published a book titled 'De Magneta' which may be considered as first scientific account on the subject. According to him, in all experiments on frictional electricity, two kind of charges are produced. An American scientiest Benjamin Franklin (1706-1790), named these charges as positive and negative. These names are followed till today.

14.3.1 Properties of Electric Charges

In electrostatics, the process of charging a body has been traced to the actual transfer of electrons. You are aware of the fact that every atom consists of nucleus around which negatively charged particles revolve in orbits. When we rub two substances against each other, we provide energy to over come friction between them. This energy is used in removing electrons from one substance and transferring them to the other. This transfer takes place from the material in which electrons are held less tightly to the material in which they are strongly attached. In this process, the material which losses electrons acquires a positive charge and the material which gains electrons acquires an equal amount of negative charge. "What is an electric charge"? It is a difficult question of physics. This is because ***"charge is better understood not by saying of what it is but by what it does".*** We may, however, define it as follows – The electric charge is a basic and characteristics property of the elementary particles such as electron, proton etc. We can identify electric charge with some properties, which are mentioned in the following text.

(a) Charges are of two kinds : There are only two kinds of charges, positive charge and negative charge. There is no third kind of charge. The positive charge in ordinary matter

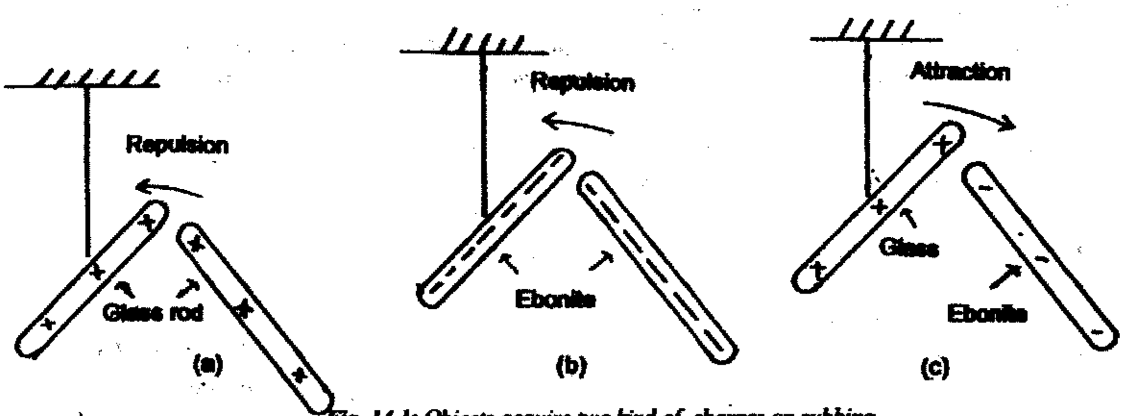


Fig. 14.1: Objects acquire two kind of charges on rubbing

is carried by proton and negative charge by electron. To show that there are two kinds of charges, let us take a glass rod, rub it with silk cloth and suspend it by a thread. If another glass rod rubbed by silk is brought near to it, as shown in Fig14.1 (a), two glass rods will repel each other. Similarly two ebonite rod each rubbed with fur repel each other Fig. 14 1(b). But a glass rod rubbed with silk attracts an ebonite rod rubbed with fur as shown in Fig14.1(c). These observations show that the charges developed on glass rod are different from the charges deveped on abonite rod.

Conventionally, the charge developed on glass rod is taken as positive and that on the ebonite rod as negative. These observations also show that ***like charges repel each other while unlike charges attracts each other.*** Electric charge is thought of as the source of electric force just as mass is the source of gravitational force.

(b) Quantization of electric charges : Any charge exists only in integral multiples of a certain minimum charge, i.e. the charge on an electron. The value of this minimum charge ($e = 1.6 \times 10^{-19}$ coulomb) is so small that the graininess of charge (or quantization of charge) does not show up in large scale experiments in real life. This property *which envisages the discrete nature of charge, ruling out its continuous nature, is known as quantization of electric charge.* According to quantization of charge, any charged body can have only an integral multiple of the magnitude of charge on an electron (also known as quanta of charge) i.e., $q = \pm ne$, where n ia an integer i.e., $n = 0, 1, 2, 3, \ldots$. The fractional value of charge lying in between these values is not possible.

All the known fundamental particles have charges that are some integeral multiples of fundamental charge i.e., charges are always $0, \pm e, \pm 2e, \pm 3e, \ldots$ etc.

(c) Conservation of electric charge : According to this property *the algebraic sum of electric charges on all the bodies in an isolated system remains constant for all times.* Thus it is clear that charge can neither be created nor can be destroyed. It can only be transfered from one body to other. As you know, when a glass rod is rubbed with silk, the amount of positive charge acquired by glass rod is exactly equal to the amount of negative charge produced on silk. The algebraic sum of charges on glass rod and silk, before and after rubbing, continues to be zero. The conservation of electric charge is one of the most fundamental properties of charge and no violation of this, has ever been observed till today, in any natural event or in the laboratory experiment. According to this property charges can be created only in equal and opposite pairs as in case of pair production i.e. production of electron and positron pair from X-rays.

Properties of electric charge

1. *Charge is a scalar quantity;*
2. *There are two and only two type of charges namely positive and negative;*
3. *Charges of same type, always repel one another while charges of opposite kind always attract one another;*
4. *Charge is additive in nature i.e., total charge of a given system is algebraic sum of all the charges present in it;*
5. *Charge is quantized;*
6. *Charge is conserved.*

INTEXT QUESTIONS 14.1

1. Can a body have a charge 0.8×10^{-19} C ? Give reason.

....

2. A glass rod is rubbed with silk cloth and it gains a positive charge of 4.8×10^{-18}C. What type and how much charge will be gained by the silk cloth ?

....

3. Give an example illustrating the conservation of electric charge.

....

14.3.2 Electrostatic Interaction between two Charges : Coulomb's Law

When an electric charge is held in the viscinity of another charge, it experiences a force. According to Coulomb's law, this force of interaction between two point electric charges is:

i) directly proportional to the product of the magnitudes of the charges;
ii) inversely proportional to the square of the distance between them; and
iii) acts along the straight line joining the two charges.

If q_1 and q_2 are two point charges separated in vacuum by a distance 'r'. Then according to Coulomb's law the force (F) between them is :

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = \frac{k q_1 q_2}{r^2} \qquad ...(14.1)$$

Where 'k' is electrostatic force constant. As shown in Fig.14.2, force 'F' acts along the straight line joining the two charges and is directed

a) towards q_1 (or q_2), if the charges are of opposite kind.
b) away from q_1 (or q_2) if the charges are of same kind.

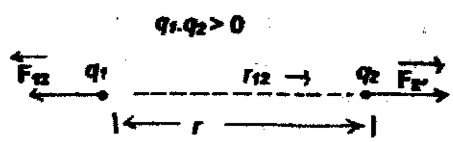


Fig. 14.2: Coulomb's Law

If F_{21} = force on q_2 due to q_1 and $\hat{r}_{12}$ = unit vector pointing from q_1 to q_2 then according to Coulomb's law.

$$F_{21} = \frac{k\, q_1\, q_2}{|r_{21}|^2} \times \hat{r}_{12}$$

Similarly, the force on q_1 due to q_2 is

$$F_{12} = \frac{k\, q_1\, q_2}{|r_{21}|^2}\hat{r}_{21}$$ (Where $\hat{r}_{21}$ is unit vector pointing from q_2 to q_1.

As we know $\hat{r}_{12} = -\hat{r}_{21}$, $\qquad \therefore F_{21} = -F_{12}$ $\qquad$ ...(14.2)

This shows that forces exerted by the two charges on each other are equal and opposite. The experimentally measured value of electrostatic force constant k is 9×10^9 Nm^2C^{-2}. In SI unit we write $k = \frac{1}{4\pi\varepsilon_0}$ where ε_0 (spoken as epsilon zero) is known as absolute electric permittivity of the free space and its value is 8.85×10^{-12} $C^2 N^{-1} m^{-2}$.

Remember strictly speaking Coulomb's law is valid only for point charges. It is true for very large distances to very small distances such as atomic distance (= 10^{-11} m or small)

You can note that the Coulomb's law of electrostatic force between two charges is quite similar to Newton's law of gravitational force between two masses as

$$F = G\frac{m_1 \; m_2}{r^2}\hat{r}$$

The *SI Unit of electric charge is Coulomb* and it is *defined as the charge which when placed at a distance of 1m from an identical charge, in vacuum, will repel it with a force of* 9×10^9 N.

14.3.3 Vector Form of Coulomb's Law

Let two point charges q_1 and q_2 be located at points A and B specified by position vectors r_1 and r_2 respectively (see Fig14.2). We then write Coulomb's law in vector form as

$$F_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2_{21}}\hat{r}_{21}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|r_{21}|^3}r_{21}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|r_2 - r_1|^3}(r_2 - r_1)$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|r_2 - r_1|^3}(r_1 - r_2)$$

$$= -F_{21}$$

Here $\hat{r}_{21} = \frac{r_{21}}{|r_{21}|} = \frac{(r_2 - r_1)}{(r_2 - r_1)}$

and $\hat{r}_{12} = \frac{r_{12}}{|r_{12}|} = \frac{(r_1 - r_2)}{(r_1 - r_2)}$

In case one of the charges is situated at the origin itself $r_1 = 0$. $r_2 = r$ (say).

Then, $F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|r|^3}r$ .... (14.3)

14.3.4 The "Dielectric Constant" of the Medium

The force between two charges q_1 and q_2 kept at a given distance "r" apart, decreases, when the charges are present in a medium other than vacuum. In case the charges are kept in a medium Coulomb's law is written as :

$$F_{21} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

The factor ε_r, turns out to be a characteristic property of the medium and is known as its ***relative permittivity*** or its ***dielectric constant.***

> The ***dielectric constant*** (or relative permitivity) of a medium may be defined as the ratio of the magntude of force between the two charges placed at a given distance apart in vacuum, to the force between the same two charges, when they are similarly placed, the same distance apart, in the given medium.
>
> $$\text{Thus, } \varepsilon_r = \frac{F(\text{vacuum})}{F(\text{medium})}$$

We often write :
Absolute permitivity $\varepsilon = \varepsilon_o \varepsilon_r$

Note that ε_r is a dimensionless constant and it has no unit.

14.3.5 Forces among many Charges and Superposition Principle

If a large number of charges $q_1, q_2, q_3 \ldots\ldots q_n$ are present in space, at position $r_1, r_2 \ldots r_n$, the force on any charge 'q_o' placed at position r due to these charges (See Fig. 14.3) is given by:

$$\mathbf{F}_o = \mathbf{F}_{o1} + \mathbf{F}_{o2} + \mathbf{F}_{o3} + \ldots\ldots\ldots\ldots\mathbf{F}_n$$

or

$$F_o = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_o\, q_i (r_i - r_o)}{|r_i - r_o|^3} \quad (14.5)$$

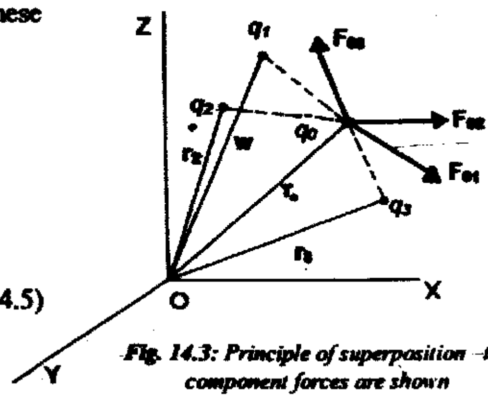


Fig. 14.3: Principle of superposition - the component forces are shown

According to the ***superposition principle***, the ***force between two charges is not altered due to the presence of the other charges.*** Coulomb's law is applicable to calculate the interaction of each pair, irrespective of the presence of other charges in the system. The total force experienced by any charge due to the presence of the rest of charges in the system can be obtained by the vector sum of all forces of interaction on the charge.
Electric forces produced by different charges combine in accordance with laws of vector addition.

14.4 ELECTRIC FIELD AND LINES OF FORCE

To explain the mechanism of interaction between two charges placed at a distance the idea of electric field is concieved. When charged body is placed in the neighbourhood of another charged body, it experiences an electrostatic force, we say that there is an electric field in the surrounding of the charged body. The value of electric field at any point is calculated in terms of electric field intensity, which is explained below.

14.4.1 Electric Field Intensity

The electric field intensity at any point is the strength of electric field at that point. It is defined as the force per coulomb of charge acting on a very small positive test charge placed at that point.

i. e. $\mathbf{E} = \frac{F}{q_o}$ NC^{-1} ... (14.6)

Here, **E** is electric field intensity (strength) and **F** is the force acting on the test charge 'q_o' at that point. *It is a vector and its direction is same as the direction of* **F**.

The use of concept of electric field is that we can calculate readily the magnitude and direction of the force experienced by any charge held at that point in the electric field i.e. $\mathbf{F} = q\mathbf{E}$. The direction of force on positive charge is the direction of electric field intensity.

Intensity of electric field as a point r due to a point charge q at the origin is given by,

$$|\mathbf{E}| = \frac{|\mathbf{F}|}{q_o} = \frac{1}{q_o} \times \frac{1}{4\pi\varepsilon_o} \frac{qq_o}{|r|^2}$$

$$= \frac{1}{4\pi\varepsilon_o} \frac{q}{|r|^2} \quad ...(14.7)$$

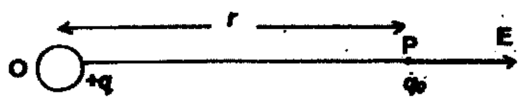


Fig.14.4 : Electric field intensity due to a point

Similarly, field due to a number of charges in space can be calculated. The force experienced by test charge q_o at any point using principle of superposition is given as,

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_o q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Thus, electric field intensity at that point

$$\mathbf{E} = \frac{\mathbf{F}}{q_o} = \frac{1}{4\pi\varepsilon_o} \sum_i \frac{q_i}{r_i^2} \hat{r}_i \quad ...(14.8)$$

$$= \frac{1}{4\pi\varepsilon_o} \left[\frac{q_1}{r_1^2} r_1 + \frac{q_2}{r_2^2} \hat{r}_2 + \frac{q_3}{r_3^2} \hat{r}_3 + ... \right]$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 + \; .... \quad ...(14.9)$$

Hence, electric field intensity at any point is the vector sum of the intensities due to each individual charge. *The principle of superposition is valid in case of electric field intensity also.*

14.4.2 Electric Lines of Force

Electric field around a charge configuration may be depicted in terms of electic lines of force. An *electric line of force, in an electric field, is the imaginary smooth curve, the tangent to any point of which gives the direction of the field at that point.*

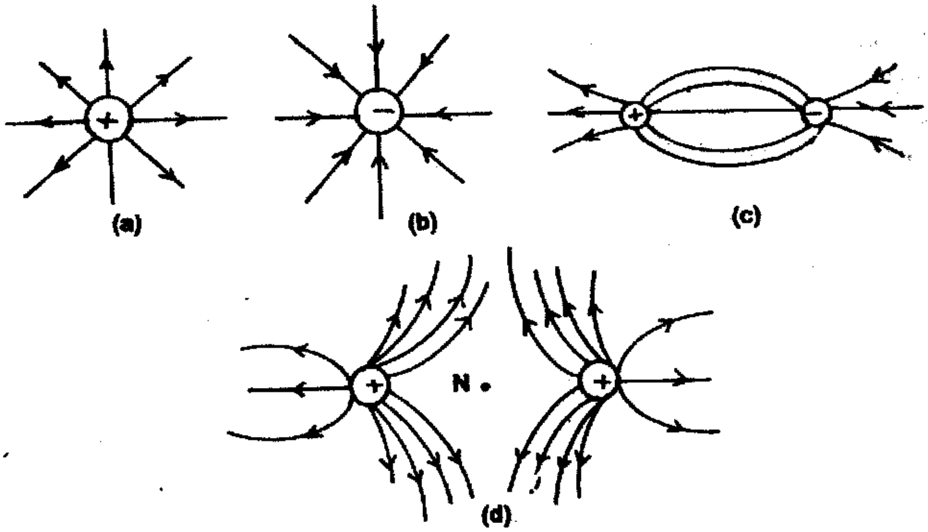


Fig. 14.5: Lines of force for different charge systems

In Fig. 14.4, lines of force of different systems are shown. Lines of force due to an isolated positive charge go straight into infinity and on the other hand due to a free negative charge, lines of force comes straight from infinity upto negative charge (Fig. 14.5 a and b). It is also clear from Fig. 14.5 (a) and (b) that for a charged sphere, the lines of forces are straight and radial and appear either emerging from or heading towards the centre of the sphere. In Fig. 14.5 (c), lines of force are drawn for a system of two equal and opposite charges. The lines of force are drawn in the field produced by two equal and similar (positive) charges in Fig. 14.5 (d). At the mid-point N of the line joining the two charges, the resultant field is zero and this point is known as ***'neutral point'***. No two lines of force can intersect each other, because if they do so, then at the point of intersection two tangents can be drawn which would mean two directions of the force at that point, which is not possible. ***The positive charges are sources of lines of force and negative charges are sinks.***

INTEXT QUESTIONS 14.2

1. *Why two electric lines of force can not intersect each other ?*

   ....

2. *Distinguish between electric field and electric field intensity at a point.*

   ....

3. *While detecting the electric field why the test charge q_o should be very small ?*

   ....

14.5 ELECTRIC DIPOLE AND DIPOLE MOMENT

An electric dipole consists of a pair of equal and opposite point charges separated by a very small distance. Several molecules such as HCl, H_2O, Ammonia etc., behave as permanent electric dipoles, because in these molecules, the centres of positive and negative charge distributions are separated by some small distance (although the net charge on the molecule remains zero). Fig. 14.6 shows an electric dipole consisting of two equal and opposite point charges ($\pm q$) separated by small distance '2ℓ'. The product of one of the charge of dipole and distance between the charges is called '***dipole-moment***' and it is usually denoted by **p**.

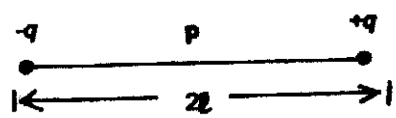


Fig. 14.6: Electric dipole and direction of electric dipole moment

$$|\mathbf{P}| = q \times 2\ell \; ; \; \mathbf{p} = 2q\ell\, \hat{\ell} \qquad ...(14.10)$$

The electric dipole-moment is a 'vector' quantity, whose direction is along the axis of the dipole pointing from negative charge to the positive charge. Its unit is 'coulomb-meter'.

14.5.1 Electric field of a dipole

There exists an electric field around an electric dipole. To calculate electric field intensity of a dipole at any point, we imagine a unit positive charge held at that point. We calculate force on this charge due to both the charges of dipole and take vector sum of the two forces. This gives us dipole field intensity at that point. In this connection, two position are important **(a)** ***electric field at a point on the axis of dipole known as end-on position and*** **(b)** ***electric field at a point on the equatorial line of the dipole known as broad-side-on position.***

(a) Field intensity at a point on the axis of dipole (end-on-position)

Consider an electric dipole AB consisting of charge $+q$ and $-q$ separated by small distance '2ℓ'. We have to determine electric field intensity **E** at a point P on the axial line of the dipole and at a distance OP $= r$ from the centre O of the dipole (See Fig. 14.7).

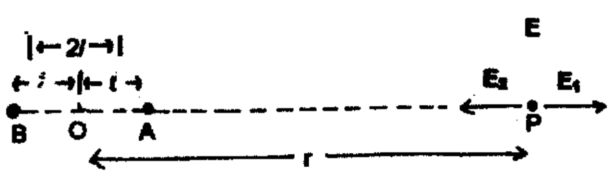


Fig. 14.7: Electric field intensity due to dipole at end-on-position

Let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the intensities of electric field at P due to the charge $+q$ and $-q$ of the dipole respectively. Obviously,

$$|\mathbf{E}_1| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)^2} \text{(In the direction AP)}$$

$$|\mathbf{E}_2| = \frac{1}{4\pi\varepsilon_o} \frac{q}{(r+\ell)^2} \text{ (In the direction PA)}$$

Thus, resultant electric field intensity **E** at the point P will be

$$|\mathbf{E}| = |\mathbf{E}_1| - |\mathbf{E}_2| \text{ (in the direction AP since } \mathbf{E}_1 > \mathbf{E}_2)$$

$$|\mathbf{E}| = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{4\ell r}{(r^2-\ell^2)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{2(2q\ell)r}{(r^2-\ell^2)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{2|\mathbf{p}|r}{(r^2-\ell^2)^2}\right] \quad \text{where } |\mathbf{p}| = 2q\ell = \text{dipole moment}$$

If 'ℓ' is very small as compared to r, then ℓ term may be neglected in the above expression. Then, electric field intensity at P due to dipole is given by

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_o}\frac{2|\mathbf{p}|r}{r^4} = \frac{1}{4\pi\varepsilon_o}\frac{2|p|}{r^3}\ \text{NC}^{-1} \qquad ...(14.11)$$

The electric field is in the direction of dipole moment i.e. along the axis of dipole from the negative charge towards the positive change.Clearly $|\mathbf{E}| \ \alpha \ \frac{\ell}{r^3}$

(b) Field intensity at a point on the equatorial line of a dipole (broad-side-on-position)

Consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a small distance AB = 2ℓ with centre at O (Fig. 14.8). Suppose that the point P is situated on the right–bisector of the dipole AB at a distance r meter from its midpoint O. Again let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the intensities of electric field at P due to the charges $+q$ and $-q$ of the dipole respectively. The distance of P from each charge is $\sqrt{r^2+\ell^2}$

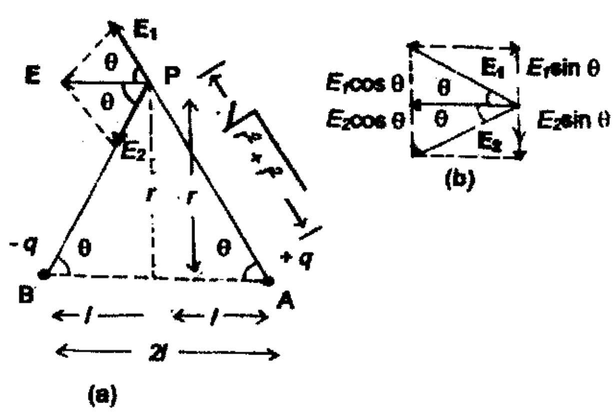


Fig. 14.8: Electric field intensity due to dipole at broad-side-on position (equatorial Line)

We, thus, have

$$|\mathbf{E}_1| = \frac{1}{4\pi\varepsilon_o}\ \frac{q}{(r^2+\ell^2)} \text{ (In the direction AP)}$$

$$|\mathbf{E}_2| = \frac{1}{4\pi\varepsilon_o}\ \frac{q}{(r^2+\ell^2)} \text{ (In the direction PB)}$$

On resolving $\mathbf{E}_1$ and $\mathbf{E}_2$, parallel and perpendicular to AB, the components perpendicular to AB ($\mathbf{E}_1 \sin\theta$ and $\mathbf{E}_2 \sin\theta$) cancel each other as they are equal and opposite. The components parallel to AB ($\mathbf{E}_1 \cos\theta$ and $\mathbf{E}_2 \cos\theta$) add up being in the same direction (see Fig. 14.9 b). Hence resultant electric field intensity at P is

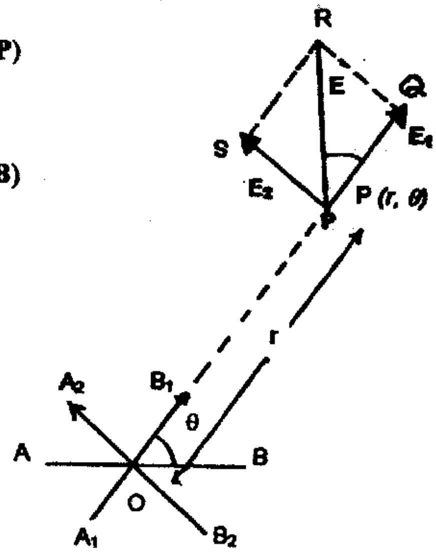


Fig : 14.9:

$$|\mathbf{E}| = |\mathbf{E}_1|\cos\theta + |\mathbf{E}_2|\cos\theta$$

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_o}\ \frac{q}{(r^2+\ell^2)}\cos\theta + \frac{1}{4\pi\varepsilon_o}\ \frac{q}{(r^2+\ell^2)}\cos\theta$$

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_o}\ \frac{2q}{(r^2+\ell^2)}\cos\theta$$

From Fig. 14.9, we get $\cos\theta = \frac{OA}{PA} = \frac{\ell}{(r^2+\ell^2)^{1/2}}$

Thus, $$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_o}\ \frac{2q}{(r^2+\ell^2)}\ \frac{\ell}{(r^2+\ell^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_o}\ \frac{2\ell q}{(r^2+\ell^2)^{3/2}} = \frac{1}{4\pi\varepsilon_o}\ \frac{|\mathbf{p}|}{(r^2+\ell^2)^{3/2}}$$

where $|\mathbf{p}| = 2q\ell$ = dipole moment

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\ \frac{\mathbf{p}}{(r^2+\ell^2)^{3/2}}$$ $\therefore$ **E is effective in opposite direction to p**

If 'ℓ' is very small as compared to 'r' then ℓ^2 can be neglected in the above relation. Then electric intensity at a point P due to dipole is.

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_o}\ \frac{|\mathbf{p}|}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_o}\ \frac{|\mathbf{p}|}{r^3} \text{ N/C} \qquad ...(14.12)$$

From these calculations, we see that for a dipole, the intensity on an axial point is twice the intensity at the same distance on the equitorial line.

14.5.2 Behaviour of Electric Dipole in a Uniform Electric Field

Let an electric dipole be placed in a uniform external electric field **E** at an angle θ with the direction of **E** (Fig. 14.10). In the field the force on charge $+q$ is $\mathbf{F} = q\mathbf{E}$ along the direction of **E** and the force on change $-q$ is $\mathbf{F}=q\mathbf{E}$ in the direction apposite to **E**. These forces being equal, unlike and parallel, form a couple which rotates the dipole in clockwise direction and tends to align it along the direction of external electric field **E**.

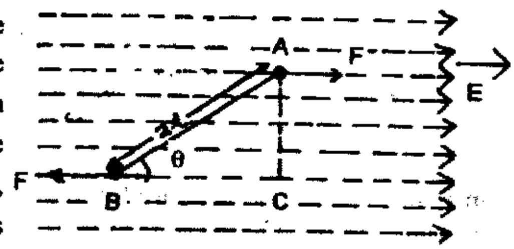


Fig. 14.10 :

Couple on an electric dipole in a uniform electric field

We know

Torque (τ) $= \text{Force} \times \text{arm of couple}$
$= F \times \overline{AC}$
$= F \times AB \sin\theta$ (From Fig 14.9)
$= (q\,E)\,(2l \sin\theta\,)$

$$\boxed{\tau = pE \sin\theta} \qquad ..(14.13)$$

In vecter notations, we can rewrite the above relation as

$$\boxed{\tau = \mathbf{p} \times \mathbf{E}} \qquad (14.14)$$

Remember, the force on the two charges of the dipole are equal and opposite, therefore net force on dipole is zero.

Case -I : We see, that in case the dipole is in the direction of external electric field, $\theta = 0$ and then $\tau_{min} = p\,E \sin(\theta) = 0$ (Minimum Value)

Case -II : Torque will be maximum when $\theta = 90°$ i.e. dipole is kept perpendicular to external electric field .

$\tau_{max} = p\,E \sin(90) = p\,E$ (Maximum Value)

INTEXT QUESTIONS 14.3

1. *A comb after running through one's dry hair attracts small bits of paper, why ? What happens when hair is wet.*

   . . . .

2. *Two -free protons and two free electrons are separated by the same distance. Compare Coulomb's force of repulsion between a pair of electron and a pair of protons.*

   . . . .

3. *Can two similarly charged balls attract each other?*

   . . . .

4. *Give two basic differences between charge and mass.*

   . . . .

5. *Answer whether the following statements are true or false?*
   a) *Electric charge is additive.*
   b) *10 Coulomb = 10 Ampere sec.*
   c) *Dipole moment is a scalar quantity.*
   d) *Electric line of force can cut only at neutral point.*
   e) *Intensity of electric field due to a dipole on an electric axial point is twice the field at the same distance on its equitorial line.*
   f) *When dipole is parallel to uniform external electric field, net force acting upon it is zero but torque will have small finite value.*

14.6 GAUSS THEOREM

Total number of lines of force passing through an area immersed in an electric field (E) is known as flux. It is usually represented by φ_E.

The electric flux through a surface is defined as the product of the area (S) and the component of the electric field normal to the surface. (E_n)

i.e., $\boxed{\varphi_E = E_n . S}$ ...(14.15)

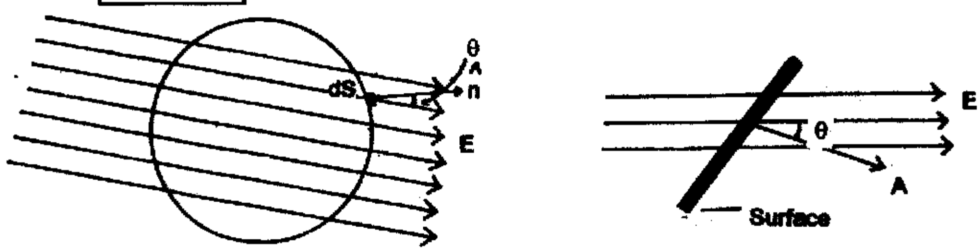


Fig. 14.11 :

Thus, electric flux through a close surface will be positive or nagative depending on whether the field lines emerge from the surface or enter into the surface.

14.6.1 Gauss Theorem in Electrostatics

Gauss's law states that total electric flux linked with a closed surface (S) in vacuum is $\frac{1}{\varepsilon_o}$ times the algebraic sum of charges (i.e. total charge) enclosed by the surfaces

i.e. $$\varphi_E = \frac{\Sigma q}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

or $$E.S = \frac{Q}{\varepsilon_o}$$ ...(14.16)

Remember location of Q inside close surface S does not affect the value of flux . There is no contribution to total electric flux due to the charges present outside the closed surface. ***The closed surface involved in this law is usually called the Gaussian surface.*** **Note** that during the application of Gauss's law, Gaussian surface is usally chosen in such a way so that the electric field intensity may have a single fixed value at every point on the surface. While selecting such a surface, we should avoid charges on S itself.

14.6.2 Important Applications of Gauss's Theorem

(a) Determination of electric field due to a point charge

Consider an isolated positive point charge q at O. To determine field at point P distant r from O imagine a spherical Gaussian surface of radius 'r' with centre O [Fig. 14.11] Let **E** be the magnitude of electric field intensity on the Gaussian surface and it is directed radially outwards. Further, the direction of vector dS representing a small area element dS on the spherical Gaussian surface is along **E** only, i. e, $\theta = 0$

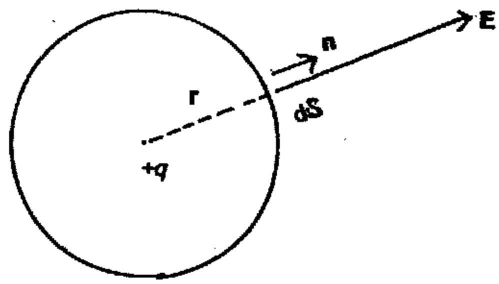


Fig. 14.12:

According to definition, flux from small area

$$\varphi_E = \mathbf{E}.\ \mathbf{S} = \mathbf{E}.\ 4\pi r^2.\cos\theta$$
$$= 4\pi r^2 \mathbf{E} \qquad ...(14.17)$$

According to Gauss's Law, total flux passing through clossed surface,

$$\varphi_E = \frac{q}{\varepsilon_o} \qquad ...(14.18)$$

from (14.17) and (14.8)

$$4\pi r^2 |\mathbf{E}| = \frac{q}{\varepsilon_o}$$

$$|\mathbf{E}| = \frac{q}{4\pi\varepsilon_o r^2}$$

$$\text{or, } \mathbf{E} = \frac{q\ \mathbf{r}}{4\pi\varepsilon_o |r|^3} \qquad ...(14.19)$$

This is the electric field entensity at any point at a distance 'r' from an isolated point charge q.

(b) Electric field intensity due to a long line of charge

A line charge is in the form of a thin charged rod with uniform linear charge density λ (charge per unit length). We have to find an expression for electric field intensity at any point P at a perpendicular distance r from the rod.

Let us consider a right circular closed cylinder of radius r and length ℓ, with the infinitely long line of charge as its axis, Fig. 14.13. The magnitude of electric field intensity **E** at every

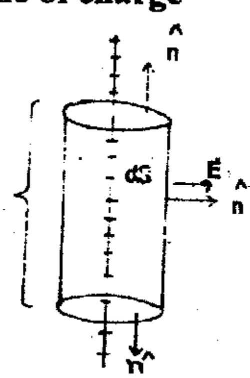


Fig. 14.13

point on the curved surface of the cylinder is the same, because all such points are at the same distance from the line charge.

Also, **E** and unit vector $\hat{n}$ normal to curved surface are in the same direction, so that $\theta = 0°$

∴ Therefor, contribution of curved surface of cylinder towards electric flux,

$$\Sigma \mathbf{E}.\, d\mathbf{S} = \Sigma\ \mathbf{E}.\hat{n}\, dS = \mathbf{E}\, \Sigma\ dS = \mathbf{E}\,(2\,\pi\, r\ell)$$

where $(2\,\pi\, rl)$ is area of the curved surface of the cylinder.

On the edges of the cylinder, angle between electric field intensity **E** and $\hat{n}$ is 90°. Therefore, these edges make no contribution to electric flux of the cylinder.

$$\varphi_E = \mathbf{E}\,(2\,\pi\, r\ell)$$

Charge enclosed in the cylinder = liner charge density × length

$$q = \lambda \ell$$

According to Gauss's theorem

$$\varphi_E = \frac{q}{\varepsilon_o}$$

$$\therefore \quad \mathbf{E}\,(2\pi r \ell) = \frac{\lambda \ell}{\varepsilon_o}$$

$$\mathbf{E} = \frac{\lambda}{(2\,\pi\,\varepsilon_o\, r)} \qquad ..(14.20)$$

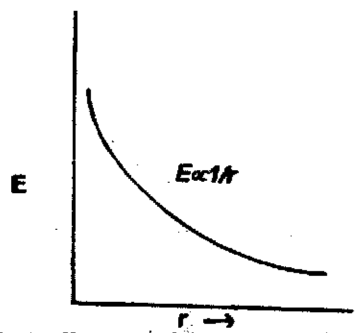


Fig. 14.14: Variation of electric field intensity with distance

Clearly, $\mathbf{E} \propto \frac{1}{r}$

The variation of electric field intenty E with distance is shown in Fig. 14.14

(c) Electric field intensity due to a uniformly charged spherical shell

Consider a thin spherical shell of radius R and centre O. Let a charge $+q$ be distributed uniformly over the surface of the shell (see Fig. 14.15). To calculate electric field intensity at any point P, where OP $= r$, imagine a sphere S with centre O and radius r. The surface of this sphere is a Gaussian surface at every point of which electric field intensity E is the same, directed radially outwards (as is unit vector $\hat{n}$, so that $\theta = 0°$).

According to Gauss's theorem

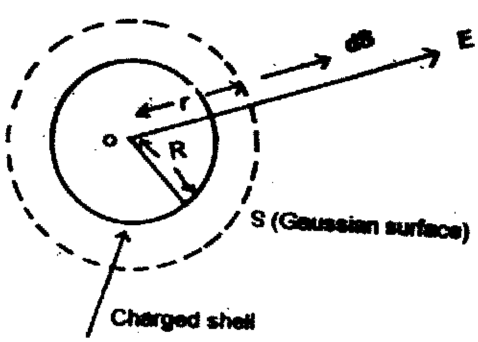


Fig. 14.15: Uniformly charged spherical shell

$$\Sigma \mathbf{E}.\, dS = \Sigma\ \mathbf{E}.\hat{n}\, dS = \frac{q}{\varepsilon_o}$$

or $$\mathbf{E}\ \Sigma\ dS = \frac{q}{\varepsilon_o}$$

$$\mathbf{E}.4\ \pi r^2 = \frac{q}{\varepsilon_o}$$

$$\mathbf{E} = \frac{q}{4\ \pi \varepsilon_o\ r^2} \qquad ...(14.21)$$

Clearly, electric field intensity at any point outside the spherical shell is such , as if the entire charge were concentrated at the centre of the shell. Let us. consider a few special cases.

Case - I: ***At a point on the surface of the shell***

$$r = R$$

$$\therefore \quad \mathbf{E} = \frac{q}{4\ \pi \varepsilon_o R^2}$$

If σ is surface denisty of charge on the shell, then

$$q = 4\pi\ R^2\ \sigma$$

$$\therefore \quad \mathbf{E} = \frac{4\pi\ R^2\ \sigma}{4\ \pi \varepsilon_o\ R^2} = \frac{\sigma}{\varepsilon_o} \qquad ...(14.22)$$

Case-II : ***If the point P lies inside the spherical shell***, then Gaussian surface is surface of a sphere of radius r ($< R$), as charge inside a spherical shell is zero. Therefore the Gaussian surface in the case encloses no charge.

i.e. $q = 0 \quad \therefore\ \mathbf{E} = 0$

i.e. electric field inside a spherical shell is always zero.

The variation of electric field intensity **E** with distance from the centre of a uniformly charged spherical shell is shown in Fig. 14.16

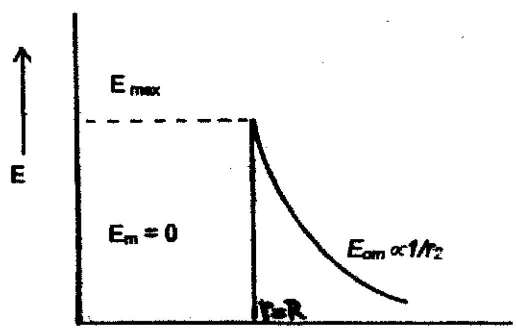
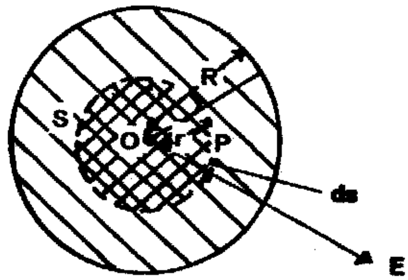


Fig. 14.16: Variation of electric field intensity with distance in the case of a charged spherical shell

(d) Electric field intensity due to a charged solid sphere

Suppose a solid sphere of radius R and centre O has uniform volume density of charge P. as shown in Fig. 14.17.

We have to calculate electric field intensity **E** at any point P. where OP = r. With O as centre and r as radius, imagine a sphere S, which acts as a Gaussian surface. At every point of S, magnitude of **E** is same, directed radially outwards.



If q' is the charge enclosed by the sphere S, then according to Gauss's law

$$\sum_S \mathbf{E}\cdot d\mathbf{S} = \sum_S \mathbf{E}\cdot\hat{\mathbf{n}}\, d\mathbf{S} = E\sum_S d\mathbf{S} = \frac{q'}{\varepsilon}$$

Fig. : 14.17:

where ε_o is electrical permittivity of the material of the sphere.

$$\therefore \quad \mathbf{E}\,(4\pi r^2) = \frac{q'}{\varepsilon} \quad \text{or} \quad \mathbf{E} = \frac{q'}{4\pi\varepsilon r^2}$$

Now, charge inside sphere S.

q' = volume of S × volume density of charge

$$q' = \frac{4}{3}\pi r^3 \times \rho$$

$$\mathbf{E} = \frac{4\pi r^3\ \rho}{3\times 4\pi\varepsilon_o\ r^2} = \frac{r\rho}{3\varepsilon_o} \quad \text{i.e} \quad \boxed{\mathbf{E} = \frac{r\rho}{3\varepsilon_o}} \qquad ...(14.23)$$

Clearly, $\mathbf{E} \propto r$.

Case -I :*At the centre of the sphere, r = 0,*

$$\therefore \quad \mathbf{E} = 0$$

Case -II: *At the surface of the sphere, r = R*

$$\mathbf{E} = \frac{R\rho}{3\varepsilon_o} \rightarrow \text{maximum} \qquad ...(14.24)$$

We have already proved that outside the sphere,

$$\mathbf{E} \propto \frac{1}{r^2} \qquad ...(14.25)$$

All these results are shown in Fig. 14.18 which represents the variation of electric field intensity **E** with distance *(r)* from the centre of uniform sphere of charge.

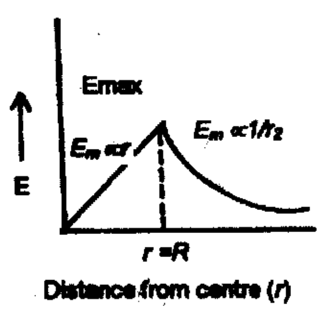


Fig. 14.18 : Variation of electric field intensity

(e) Electric field intensity due to a thin infinite sheet of charge

Consider a thin, *infinite plane sheet* of charge. Let σ be the surface density of charge on the sheet (i.e. charge/area). We have to calculate electric field intensity **E** at any point P, distant *r* from the sheet (See Fig. 14.19)..

From symmetry, we find that *E* on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.

To calculate **E**, at a point distant *r* from the sheet let us imagine a cylinder of cross-sectional area dS and length 2*r* piercing through the sheet. At the two cylindrical edges P and Q, **E** and $\hat{n}$ are parallel to each other, Fig. 14.19.

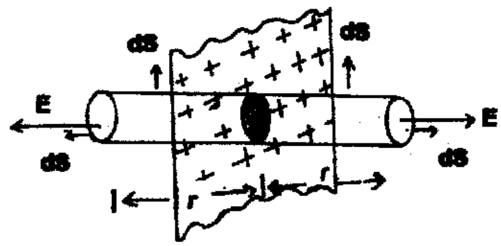


Fig. : 14.19: Electric field due to a this infinite sheet of charge

$\therefore$ Electric flux over these edges = $2\mathbf{E}.\,\hat{\mathbf{n}}dS = 2\mathbf{E}\,dS$

On the curved surface of the cylinder, **E** and $\hat{\mathbf{n}}$ are $\perp$ to each other. Therefore to, contribution to electric flux is made by the curved surface of the cylinder.

$\therefore$ Total electric flux over the entire surface of the cylinder = $2\mathbf{E}\,dS$

Total charge enclosed by the cylinder = $\sigma\,dS$

According to Gauss's law in electrostatics,

$$2\mathbf{E}\,dS = \frac{q}{\varepsilon_0} = \frac{\sigma\,dS}{\varepsilon_0}$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0} \qquad ...(14.26)$$

We observe that **E** is independent of *r*, the distance of the point from the plane charged sheet.

Further, it is clear that if the sheet carries positive charge (i.e., $\sigma > 0$), **E** is directed away from both sides of the thin plane sheet and vice-versa.

If the infinite plane sheet has *uniform thickness*, the surface density of charge σ is uniform and same on both the surfaces of the sheet.

The electric field intensity at any point P due to each surface is $\mathbf{E}_1 = \mathbf{E}_2 = \sigma/2\varepsilon_0$. Both $\mathbf{E}_1$ and $\mathbf{E}_2$ are perpendicular to the plane of the

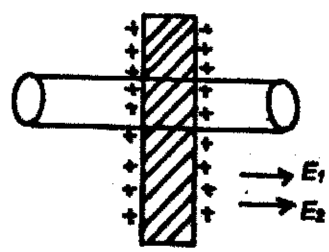


Fig. 14.20: Electric field intensity due to each surface

sheet and directed away, Fig. 14.20. Therefore, according to superposition principle, net electric field intensity at P due an ***infinite plane sheet* of *uniform thickness is,***

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}} \quad ...(14.27)$$

Note: That the sheet is supposed to be of iinifinite size so that the 'edge effect' due to distortion of electric field at the edges (of a finite sheet) can be ignored.

(f) Electric field intensity due to two thin parallel infinite sheets of charge

Let A and B be two thin infinite plane charged sheets held parallel to each other as shown in Fig. 14.21. Suppose σ_1 is uniform surface density of charge on *A and* σ_2 is uniform surface density of charge on *B.*

We use the equation obtained above to write the field intensity for each sheet and then apply superposition principle to calculate the net field intensity in the three regions. As a matter of convention, a field pointing from left to right is taken as positive and the one pointing from right to left is taken as negative.

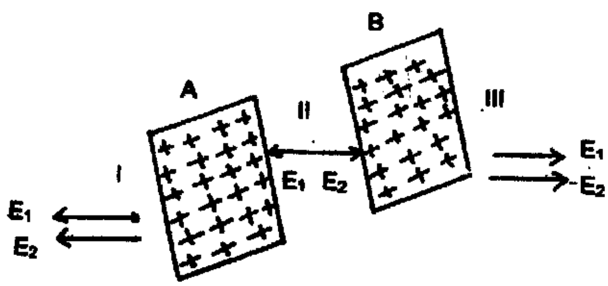


Fig. 14.21: Electric field intensity due to two thin parallel sheet of charge

We assume that $\sigma_1 > \sigma_2 > 0$.

Therefor in region I

$$\therefore \quad \mathbf{E}_I = -\mathbf{E}_1 - \mathbf{E}_2$$

$$\mathbf{E}_I = \frac{-\sigma_1}{2\varepsilon_o} - \frac{\sigma_2}{2\varepsilon_o} = -\frac{1}{2\varepsilon_o} \times (\sigma_1 + \sigma_2) \quad ...(14.28)$$

Similarly, **in region II**

$$\mathbf{E}_{II} = \mathbf{E}_1 - \mathbf{E}_2 = \frac{\sigma_1}{2\varepsilon_o} - \frac{\sigma_2}{2\varepsilon_o} = \frac{\sigma_1 - \sigma_2}{2\varepsilon_o} \quad ...(14.29)$$

and, **in region III**

$$\mathbf{E}_{III} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma_1'}{2\varepsilon_o} + \frac{\sigma_2}{2\varepsilon_o} = \frac{1}{2\varepsilon_o} (\sigma_1' + \sigma_2) \quad ...(14.30)$$

Special cases :

Suppose $\sigma_2 = -\sigma_1$ and $|\sigma_1| = |\sigma_2|, = \sigma$

i.e. two thin infinite plane sheets with equal and opposite uniform surface densities of charge are held parallel to each other.

From (14.28), $E_I = 0$
From (14.30), $E_{III} = 0$

From (14.29), $E_{II} = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$ = Constant.

Thus, field intensity in between such sheets having equal and opposite uniform surface densities of charge become constant i.e. a uniform electric field is produced. Also **E** does not depend upon the distance between the thin sheets. This is how uniform electric fields are produced in practice.

INTEXT QUESTIONS : 14.4

1. *Write the unit of electric flux.*

   ....

2. *At a point on the surface of the shell, what is the relation between electric field intensity due to a uniformally charge spherical shell ?*

   ....

3. *What is a Gaussian surface?*

   ....

4. *State whether the following statements are true or false.*

   i) *The normal component of electric field is positive when the electric field is out ward from the surface.*
   ii) *Location of charge 'Q' inside the closed surface decides the value of flux.*
   iii) *Electric field intensity due to a long line charge is independent of the distance "r".*
   iv) *For the infinite sheet the "edge effect" can be ignored.*
   v) *Electric field intensity inside a uniform solid sphere of charge is always zero.*

14.7 WHAT YOU HAVE LEARNT

- Due to friction, glass rod can be electrified and this was known to Greek philosopher. Thales as early as 600 B.C.
- Electric Charges are of two kinds.
- Charge is conserved and quantized. It is additive in nature.
- Coulomb's law state that the force of electrostatic interaction between two charges is directly proportional to the product of two charges and inversely propostional to the square of distance between them.
- Coulomb's law of electrostatic is quite similar to Newton's Law of gravitation.
- The "dielectric constant" of the medium is defined as the ratio of megnitude of force between the two charges placed at a given distance apart in vacuum, to the force between the same two charges when they are similarly placed, same distance apart in given medium.
- Electric forces produced by different charges combined by vector addition.
- Electric lines of forces and their properties.
- The positive charge are sources of lines of force and negative charges are sinks.
- To establish relation between lines of force and electric field intensity.
- About an electric dipole and related electric dipole moment.
- To derive the expression for the electric field intensity due to dipole at any (a) axial point (b) equitinal point.

- Net force on the two charges of dipole when placed in uniform electric field (E) is zero but torque $\tau = p \times E$ where p is dipole-moment.
- Flux ($d\varphi_E$) of electric field **E** through small area dS is $\varphi_E = \mathbf{E}.d\mathbf{S}$ where
- How to use Guass's law to obtain the value of electric field due to a long line charge. (ii) Point charge (iii) Charged shell (iv) Sphere of charge. (v) plane sheet of charge.

14.8 TERMINAL QUESTIONS

1. If a brass sheet is placed between two charges placed at a certain distance apart, what happens to the force between them ?

2. How many electrons should be transferred from flannel to the ebonite rod to produce a psitive charge of 3.2×10^{-7} C on flannel ?

3. State Coulomb's law in electrostatics. Explain the vector form of Coulomb's law.

4. What are the limitations of Coulomb's law ?

5. What are dielectrics ? What happens to the electrostatic force between two point charges if the medium between them is filled with a dielectric medium of dielectric constant ?

6. What do you mean by electric lines of force ? Give the characteristic properties of electric lines of force.

7. Draw the lines of force of
   (i) a point positive charge, (ii) a positively charged sphere,
   (iii) a electric dipole, (iv) a pair of positive charges.

8. What is an electric dipole ? Find an expression for the dipole moment.

9. When are the electric lines of force parallel to each other ?

10. Two point charges q_1 and q_2 are 3m apart and their combined charge is 20 μC. If one repels the other with a force of 0.075N, what are the two charges ?

CHECK YOUR ANSWERS

Intext Questions 14.1

1. No, because this charge is equal to half of the charge on an electron, which according to quantization of charge, is not possible.

2. The silk cloth will gain a negative charge of 4.8×10^{-18} C.

3. When a glass rod is rubbed with silk cloth, both get equal and opposite charge. Thus, the total charge before and after rubbing remains uncharged i.e. zero. In other words charge remains conserved.

Intext Questions 14.2

1. If they do so, the tengents at two points of intersection will show directions of electric field which is not possible.

2. Electric field is the region around a charge in which a test charge can be influenced, where as the electric field intensity is the value of field at that point.

3. Otherwise, there may be an electric field due to test charge itself.

Intext Questions 14.3

1. This is because comb gets charged by friction. If the hair is wet charge due to friction will not develop.

2. The forces are same because each proton and electron carries same magnitude of charge.

3. Yes, when charge on one is much larger than the charge on other.

4. For mass, law of conservation as well as quantzation principle, not applicable.

5. a) True (b) True (c) False (d) False (e) True (f) False

Intext Questions 14.4

1. Nm^2C^{-1}

2. $\varepsilon = \dfrac{\sigma}{\varepsilon_o}$

3. (a) True, (b) False, (c) False, (d) True, (e) False.