15

ELECTRIC POTENTIAL AND CAPACITORS

15.1 INTRODUCTION

We have already learnt in the previous lesson that an electric field around a given charge can be represented by a vector quantity E, known as intensity of electrostatic field. It is also possible to describe the same electric field by a scalar quantity called electric potential V as electric field is a conservative field. These two physical quantities E and V are closely related to each other and any one of these can be used to describe the electric field.

In this lesson we will learn about electric potential and electric potential difference. We will calculate electric potential due to point charge and electric dipole. We will establish relation between E and V. We will also learn about capacitors– their working principle and grouping. Beside these we will talk about dielectric materials and their role in capacitors.

15.2 OBJECTIVES

After studying this lesson, you should be able to :

- *describe the meaning of electric potential and electric potential difference;*
- *derive electric potential due to point charge and due to electric dipole;*
- *explain electric potential energy possessed by a charge in an electric field;*
- *understand equipotential surfaces and their properties;*
- *evaluate potential difference from electric field and derive a relation between electric field intensity and electric potential gradient;*
- *explain the behaviour of conductors in electrostatic field;*
- *define capacity and its unit and describe the principle of capacitors;*
- *explain the capacitance of parallel plate capacitor;*
- *find the equivalent capacitance in grouping of capacitors and identify the advantage of grouping;*
- *calculate energy stored in a capacitor and mention the nature of energy;*
- *list the properties of dielectric materials and explain the process of polarisation of dielectric material in an electric field;*
- *explain the role of dielectric in increasing the capacitance of a capacitor.*

15.3 ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

When a charged particle is made to move in an electrostatic field, against the force of field, work is to be done by the external agency. This work is stored up as potential energy of the charge in accordance with the the law of conservation of energy. An electric charge placed at any point in any electric field, thus, has a potential energy which is a function of its position. We can, therefore, visualize a scalar function of position, specifying the potential energy per coulomb of charge in the field, which we call as ***potential***. Different points, in the electric field, may have different potentials and then if a positively charged particle is placed in an electric field, it will tend to move from higher to lower potential to minimize its potential energy. We will see in the next lesson how this concept of potential difference leads to current flow in electric circuits.

The electric potential at any point in an electric field is equal to the work done against the electric force in moving a unit positive charge from outside the electric field to the given point in electric field. Electric potential is a *scalar quantity* as it is related to work done. The potential at a point is taken as positive when work is done against the field by the positive charge but as negative, when work is done by the electric field in moving the unit positive charge from infinity to the point in the field.

Consider two points A and B in electric field (See Fig. 15.1). If a test charge q_o is moved from the point A to the point B along any path by an external force, assuming that the charge q_o does not disturb the pre-existing field E, the amount of work done in moving the charge from A to B by the external force is given by.

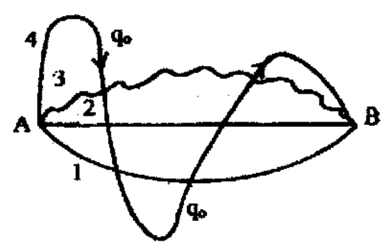


Fig.15.1:Work done between point A and B (W_{AB})

$$W_{AB} = q_o(V_B - V_A) \quad \text{....(15.1)}$$

Here, V_A and V_B are the potentials at points A and B respectively.

Thus, potential difference between point A and B, will be

$$V_{AB} = V_B - V_A = \frac{W_{AB}}{q_o} \quad \text{....(15.2)}$$

An electric potential difference is said to exist between two points in an electric field, if work is done against the electric force in moving the positive test charge from one point to the other. It must be remembered that this workdone in moving the charge from one point to other point is independent of the path. Due to this reason, the electric field is called a ***conservative field.*** SI unit of potential and potential difference is ***volt.*** The C.G.S. unit of potential and potential difference is ***stat volt*** (or e.s.u.)

1 stat volt (e.s.u.) = 300 volt.
1 volt = 1 joule/coulomb.

> If one joule of work is done in taking a test charge of 1 coulomb from one point to the other in an electric field, then the potential difference between these points will be 1 volt.
>
> If one joule of work is done in bringing a test charge of 1 coulomb from infinity to a point in the field, then potential at that point is 1 volt.

15.3.1 Potential at a Point dut to a Point Charge

Suppose we have to calculate electric potential at any point P due to a single point charge $+q$ situated at O (See Fig. 15.2), where OP = r.

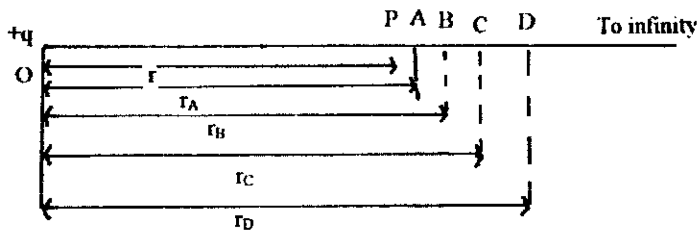


Fig. 15.2: Calculation of electric potential at P

Due to the point charge, electric intensity at 'P' will be .

$$E_p = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2} \quad ..(15.3)$$

Similarly, the electric field at point A will be

$$E_A = \frac{1}{4\pi\varepsilon_o} \frac{q}{r_A^2} \quad ...(15.4)$$

Considering point P and A to be very close to each other the average field between these points P and A may be taken as

$$E_{Ap} = \sqrt{E_A \times E_p} = \sqrt{\frac{1}{4\pi\varepsilon_o} \frac{q}{\ ^2} \times \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2}}$$

$$= \frac{1}{4\pi\varepsilon_o} \frac{q}{r_A r} \quad ....(15.5)$$

$\therefore$ Force on a charge q_o over this region will be $F_{AP} = q_o E_{AP} = \frac{1}{4\pi\varepsilon_o} \frac{q q_o}{r_A r}$ .....(15.6)

$\therefore$ Work done in moving charge q_o from A to P

$$W_{Ap} = F_{Ap} \times r_{AP} = \frac{1}{4\pi\varepsilon_o} \frac{q q_o}{r_A r} \times (r_A - r)$$

$$= \frac{q q_o}{4\pi\varepsilon_o} \times \left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad ....(15.7)$$

Similarly, work done in moving this charge from B to A

$$W_{BC} = \frac{q q_o}{4\pi\varepsilon_o} \times \left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad ....(15.8)$$

and work done in morning the charge from C to B i.e. $E_{CB} = \frac{q q_o}{4\pi\varepsilon_o} \times \left(\frac{1}{r_B} - \frac{1}{r_C}\right)$

and so on. The total work done in moving the charge from infinity to point P will be

$$W = \frac{q\,q_o}{4\pi\varepsilon_o} \times \left(\frac{1}{r} - \frac{1}{r_A} + \frac{1}{r_A} - \frac{1}{r_B} + \frac{1}{r_B} - \frac{1}{r_C} + \text{———} + \text{——} + \frac{1}{\infty}\right)$$

$$= \frac{q\,q_o}{4\pi\varepsilon_o} \quad \frac{1}{r} - \frac{1}{\infty}$$

$$= \frac{q\,q_o}{4\pi\varepsilon_o r} \qquad \text{....(15.9)}$$

In accordance with the definition of potential,

$$V_p = \frac{W}{q_o} = \frac{1}{4\pi\varepsilon_o} \quad \frac{q}{r} \text{ volts} \qquad \text{...(15.10)}$$

Note: If there are several charges of magnetude $q_1, q_2, q_3 \ldots$ .etc. then the electric potential at a point is the scalar sum of the potentials due to each charge i.e.

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots \qquad \text{....(15.11)}$$

$$\text{or} \quad V = \sum_{i=1}^{\infty} \frac{q_i}{4\pi\varepsilon_o\, r_i}$$

We see that $V \propto \frac{1}{r}$

It is positive or negative depending upon whether q is positive or negative.

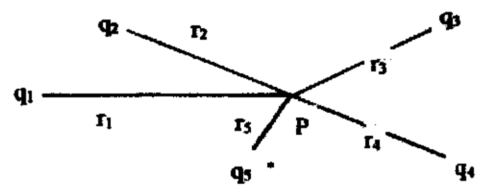


Fig. 15.3 : Potential at p due to several point charges

15.3.2 Potential at a Point due to an Electric Dipole

Let us consider an electric depole consisting of two equal and opposite point charges $-q$ at A and $+q$ at B, separated by small distance '$2l$' with centre at O. Suppose we have to calculate potential at any point P (whose polar coordinates are (r, θ) i.e. OP = r and $\angle$ BOP = θ as shown in Fig 15.4. Here AP = r_1 and BP = r_2

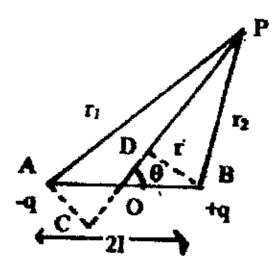
Potential at P due to the charge at A :

$$V_1 = \frac{1}{4\pi\varepsilon_o} \times \frac{(-q)}{r_1}$$

Potential at P due to the charge at B:

$$V_2 = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r_2}$$

Total potential at P due to both the charges of dipole :



$$V = V_1 + V_2$$

Fig. 15.4 : Potential at P due to dipole AB

$$V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

From Δ BOD, we get OD $= \ell\cos\theta$. Similarly from Δ OÁC we get OC $= \ell\cos\theta$
From the Fig 15.4, we get, approximately

$$r_1 = r + \ell\cos\theta$$
$$r_2 = r - \ell\cos\theta$$

because 'ℓ' is very small compared to r. Using these results in above equation, we get

$$V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-\ell\cos\theta)} - \frac{1}{(r+\ell\cos\theta)}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{2\ell\cos\theta}{(r^2 - \ell^2\cos^2\theta)}\right]$$

$$\simeq \frac{q\times 2\ell\cos\theta}{4\pi\varepsilon_o r^2} \quad \text{as } \ell << r$$

or $$V = \frac{p\cos\theta}{4\pi\varepsilon_o r^2} \qquad \text{....(15.12)}$$

Where, p = dipole moment $= q\times 2\ell$

Special Cases

Case-I: When the point P lies on the axial line of the dipole on the side of positive charge, then $\theta = 0$ and $\cos\theta = 1$

$$\boxed{V_{axis} = \frac{p}{4\pi\varepsilon_o r^2}} \qquad \text{...(15.13)}$$

Case - II: When the point lies on the axial line of the dipole on the side of negative charge, then $\theta = -180°$ and $\cos\theta = -1$

$$\boxed{V_{axis} = \frac{-p}{4\pi\varepsilon_o r^2}} \qquad \text{...(15.14)}$$

Case - III: When the point P lies on the equatorial line of the dipole, then $\theta = 90°$ and $\cos\theta = 0$

$$\boxed{V_{equatorial} = 0} \qquad \text{...(15.15)}$$

i.e. electric potential due to a dipole is zero at every point on the equatorial line of dipole.

Note : ***Equiputiontial surfac*** is that surface at every point of which electric potential is same. No work is done in moving a charge from one point of equipotential surface to the other. Electric field intensity is always perpendicular to such surfaces. A plane surface normal to the dipole axis and passing through its equitorial line is an example of equipotential surface as potential is zero at all the points on the surface.

15.3.3. Electric Potential Energy of a System of Point Charges

The electric potential energy is the energy possessed by a system of point charges by virtue of their position in the electric field. When charges are infinite distance apart, they do not interact as their fields do not extend upto each other. Under such situation, their potential energy is zero because no work is done in moving a charge placed at infinity. If we want to assemble a charge system i.e. bring charges near to each other, then work will be done. This work is stored in the form of potential energy in the system of these charges. This is called the *electric potential energy of the charge system*. Hence, we can define potential energy of a system of point charges as the total amount of work done in bringing the various point charges of the system, to their respective positions from infinitely large mutual separations.

Suppose, a point charge q_1 is held at a point P_1 with position vector r_1 in space. Another point charge q_2 is at infinity. This is to be brought to the point P_2 with position vector r_2 where $P_1P_2 = r_{12}$ as shown in Fig. 15.5. We know that electric potential at P_2 due to a charge q_1 at P_1 is,

$$V_2 = \frac{1}{4\pi\varepsilon_o}\,\frac{q_1}{|\mathbf{r}_{12}|}$$

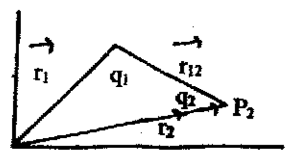


Fig. 15.5 : Potential energy of two charges

From the definition of potential, workdone in bringing charge q_2 from ∞ to point P_2 is

W= (Potential at P_2) × value of charge

$$= \frac{1}{4\pi\varepsilon_o}\,\frac{q_1}{|\mathbf{r}_{12}|} \times (q_2)$$

This work done (i.e. energy spent in doing work) is stored in the system of two point charges q_1 and q_2 in the form of electric potential energy U. Thus,

$$U = W = \frac{q_1 \times q_2}{4\pi\varepsilon_o |\mathbf{r}_{12}|} \qquad \text{....(15.16)}$$

In case of two charges of same sign, work is done against repulsive force to bring them closer and hence, electric potential energy of system increases. Conversely, in seperating them from each other, work is done by the field, as a result potential energy of the system will decrease in this process. If charges are of opposite sign i.e. one is positive and other is negative. The potential energy of the system decreases in bringing the charges near to each other while increases in separating them from each other.

Note (1): In case of a system consisting of point charges q_1, q_2, q_3 .... q_n at position vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$, ... $\mathbf{r}_n$ respectively.

$$U = \frac{1}{4\pi\varepsilon_o} \sum_{\substack{\text{all pair} \\ j \neq i}} \frac{q_i q_j}{|\mathbf{r}_{ij}|} \qquad ...(15.17)$$

(2) Potential energy of an electric dipole of moment **p** in the uniform electric field (**E**) when it makes an angle θ, is

$$\boxed{U_\theta = -\mathbf{p}\mathbf{E}\cos\theta} \qquad ....(15.18)$$

15.3.4. Relation between Electric Field and Potential Gradient

Consider any two points A and B in a uniform electric field **E**, separated by a small distance AB = d*r*.

By defination, potential difference (d*V*) between A and B = Workdone in moving unit positive test charge from A to B.

i.e. dV = (Force on unit positive charge) × (AB)

$= \mathbf{E}.(dr) = (\mathbf{E})(dr)\cos 180°$

$= -\mathbf{E}.dr$

or $|\mathbf{E}| = -\dfrac{dV}{|d\mathbf{r}|}$

or $$\mathbf{E} = \frac{-dV}{dr} \qquad ...(15.19)$$

{Negative sign indicates that work is done against the electric field.}

Hence, at any point in an electric field, intensity is equal to negative rate of change of potential with distance (called ***potential gradient***) at that point in the direction of field. Remember electric potential is a scalar quantity but electric potential gradient is a vector as it is numerically equal to electric field intensity.

From the above relation, for uniform electric field :

$$\mathbf{E} = \frac{V_A - V_B}{d} \qquad ...(15.20)$$

Here V_A and V_B are potentials at point A & B respectivly separated by a distance '*d*' meter.

INTEXT QUESTONS 15.1

1. *How is force related to charge and electric field intensity?*

 ....

2. *If a point charge be rotated in a circle of radius r around a charge q, what will be the work done ?*

 ....

3. *The electric potential V is constant in a region. What can you say about the electric field E there ?*

. . . .

4. *If electric field intensity is zero at a point, will the electric potential be necessarily zero at that point.*

. . . .

5. *Can two equipotential surfaces interseted ?*

. . . .

15.4 CAPACITANCE

On the basis of charge conduction, substances can be grouped into two classes namely conductors and insulators.In solids the conduction of electricity usually takes place due to free electrons where as in case of fluids it is due to ions. Substances that have free charge carriers through which electric currents can be established by the application of suitable electric field are called ***conductors***. Metals are good examples of conductors. Substances that have no free charge carriers are called ***insulators***. The common insulators are ebonite, glass, quartz, mica etc. Insulators are also called ***dielectrics***. The substance which have electrical conductivity between conductors and insulators are called ***semiconductors***. The ratio of electric conductivity between good conductors and good insulators is of the order of 10^{20}.

15.4.1 Behaviour of Conductors in an Electrostatic Field

As we have learnt, conductors contain free electrons and these electrons move within the conductor, when an electric field is applied. When the conductor is placed in an electric field, there is drift of electrons and this drift continues till the electric field produced due to redistribution of charge (because of drift of electrons) is equal and opposite to the applied electric field. Thus, the field within the conductors will be zero irrespective of the shape of the conductor. In most of the metals equilibrium is reached in a very short time of the order of 10^{-19} second. After equilibrium is reached, the complete interior of conductor constitutes an equipotential volume as net electric field in the interior of conductor is zero. Net charge in the interior of conductor is zero and there is no net charge density within the conductor. The whole charge always resides on the outer surface of a conductor. Therefore, surface of the conductor is an equipotential surface and there is no electric field along the surface at any point of the conducter. The electric field will be normal to the surface at any point. Thus, there is no electric field inside the conductor. Field out side the conductor can exist. This property of conductor is used ***in electrostatic shielding. Electrostatic shielding/screeing is the phenomenon of protecting a certain region of space from electric field.*** To protect delicate instruments from external electric fields, we enclose them in hollow conducters. Such hollow conductors are called ***Faraday Cages.*** These need not to be earthed. That is why in a thunder storm accompanied by lightning, it is safer to be inside a car or a bus than to be in open. The metallic body of the car or bus provides electrostatic shielding to you from the lightning.

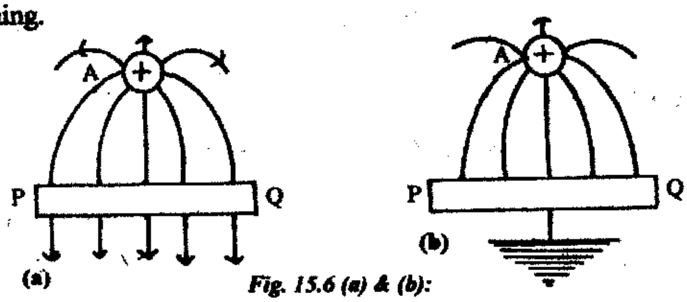


Fig. 15.6 (a) & (b):

Further, an earthed conducter PQ can also act as a screen against the electric field. You can understand it from the following [Fig. 15.6 (a) & (b)]. When PQ is not earthed. field of charged body A due to electrostatic induction continues beyonds PQ [See Fig. 15.6 (a)]. However, when PQ is grounded, the induced + charges flows to earth and the field in the region beyond PQ disappears [Fig. 15.6 (b)]. Usually high voltage generators /sources are enclosed in earthed cages. This would prevent the electrostatic field of the generater from spreading out of the cage.

15.4.2 Electrical Capacitance

Electrical Capacitance of a conductor is a measure of the ability of the conductor to store charge on it. If the charge on a conductor is gradually increased, its potential also increases and at any instant the charge (q) given to conductor, is directly proportional to its potential (V) i.e.

$$q \propto V \quad \text{or} \quad q = CV$$

or
$$C = \frac{q}{V} \qquad \text{...(15.21)}$$

Where C is constant of proportionality and is called ***capacity or capacitance of the conductor.*** The value of C *does not depend upon the material of conductor but depends upon its shape and size. It also depends upon the nature of medium in which the conductor is located.*

*The **capacitance** of conductor is defined as the amount of charge that has to be given to it to raise its potential by unity . The S.I. unit of C is farad.* In the above relation, if q=1 coulomb and V=1 volt, then C = 1 farad. Hence, capacitance of a conductor is said to be one farad, when a charge of one coulomb raises its potential through one volt.

Smaller unit of capacitance are :

1 micro farad (μF) = 10^{-6} farad.

1 micro micro farad (μμF) or picofarad (pF) = 10^{-12} farad.

15.4.3 Capacitance of Spherical Conductor

A sphere of radius r is given a charge q. Let the potential of sphere be V.

Then

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

We know capacitance $= \dfrac{\text{Charge}}{\text{Potential}}$

or
$$C = \frac{q}{q/4\pi\varepsilon_o r} = 4\pi\varepsilon_o r = \frac{r}{9\times 10^9} \qquad \text{...(15.22)}$$

It gives that $C \propto r$

Thus, we learn that the capacitance of a spherical conducter in farads is numerically equal to its radius devided by 9×10^9, where radius is taken in metre.

Example 15.1: *Calculate the earth's capacitance. taking it as a sphere of radius = 6400 km.*

Solution :

We know $C = 4\pi\varepsilon_o r = \dfrac{64 \times 10^{6}}{9 \times 10^{9}} = 0.7711 \times 10^{-13}$ F

or farad = 711 μ.F.

This shows that farad is too big a unit of capacitance as earth's capacitance is very large.

15.4.4 Capacitor

A capacitor consists of two conductors, one charged and the other usually earthed. ***The principle of a capacitor is to increase the capacitance of a conductor.***

To understand the principle of a capacitor, let us consider an insulated metal plate A. Some positive charge (q) is given to it, till its potential (V) becomes maximum. No further charge can be given as it would leak out. The capacitance of this conducter is q/V.

Now consider, another insulated metal plate B held near the plate A. By induction, negative charge is produced on the near face of B and equal positive charge develops on the farther face of B. The induced negative charge tends to decrease the potential of A and induced positive charge tends to increase the potential of A. When the plate B is earthed, the induced positive charge on B being free, flows to earth (Fig.15.7). But negative charge will stay as it is bound to positive charge on A. Due to this induced negative charge on B, potential of A is greatly reduced increasing its capacity.

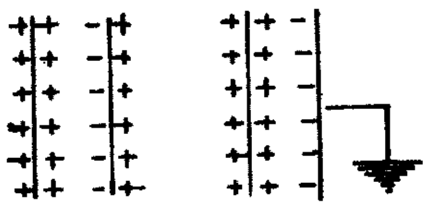


Fig. 15.7

Hence, we conclude that the capacitance of an insulated conductor is increased considerably by bringing near it an uncharged earthed conductor. This is the principle of the capacitor. Capacitors are used for storing large amounts of electric charge and hence electrical energy in a small space, for a small interval of time.

15.4.5 Capacitance of a Parallel Plate Capacitor

This is the type of capacitor which is used most commonly. It consists of two metal plates each of area A and held parallel to each other, a small distance "d" apart. Plates are seprated by an insulating medium like air, paper, mica, glass etc. One of the plate is connected to ground and other is insulated (See Fig. 15.8). When a charge $+q$ is given to the insulated plate, a charge $-q$ is induced on the inner face of nearby placed other plate and $+q$ is induced on the outer face of this plate. This plate is earthed, as explained previously. $+q$ charge being free, flows to earth. Due to $+q$ charge on one plate and $-q$ charge on other plate, an electric field intensity E will set up between the plates. When the separation 'd' is small, fringing or distortion of electric field at the boundaries of the capacitor can be neglected.

We know, if σ is surface charge density on either plate, then the electric field intensity between the plates is.

$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$ (Where V is potential difference between the plates)

Here, $\sigma = \frac{q}{A} = \frac{\text{Charge on plate}}{\text{Area of plate}}$

$$V = \frac{q}{A}\frac{d}{\varepsilon_0}$$

As we know, C = capacity $= \frac{q}{V}$

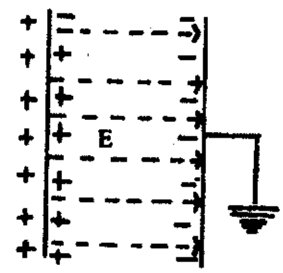


Fig. 5.8 : Parallel plate capacitor

Hence, the capacity 'C' of parallel plate capacitor separated by d in vacuum is given by,

$$C_0 = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$$

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{...(15.23)}$$

where $\varepsilon_0 = 8.86 \times 10^{-12}$ Fm^{-1}. It is clear from this relation that to obtain high capacitance,

(a) 'A' - area of plate should be large
(b) 'd' - separation between plates should be small .

Note : When the plates of capacitor are separated by some dielectric material of permittivity ε_r, other then air or vaccum, then the capacitance of parallel plate capacitor is :

$$C = \frac{\varepsilon A}{d} \quad \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Here, ε_r = relative permitivity

$$= \varepsilon / \varepsilon_0$$

Hence, $C = \varepsilon_r \frac{\varepsilon_0 A}{d} = \varepsilon_r C_0$ ...(15.24)

or $C = KC_0$

Where K is known as ***dielectric*** constant of medium. The capacitance of dielectric filled parallel plate capacitor becomes K times the capacitance with air or vacuum as dielectric.

15.4.6. Relative Permittivity or Dielectric Constant

According to Coulomb's law, the force of interaction between two charges q_1 and q_2 separated by a distance r in vacuum is,

$$F_v = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \qquad ...(15.25)$$

Where ε_o is absolute permittivity of free space or vacuum.

If same charges are held at the same distance in a material medium, then the force of interaction between them is.

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad ...(15.26)$$

From these equations we get (on dividing)

$$\frac{F_v}{F_m} = \frac{\varepsilon}{\varepsilon_o} = \varepsilon_r \text{ or } K \qquad ...(15.27)$$

where ε_r or K is relative permittivity or dielectric constant of the medium. As we notice, it is the ratio of absolute permittivity of the material medium to the absolute permittivity of free space. We can also define the dielectric constant of a medium as the ratio of the electrostatic force of interaction between two given point charges held at certain distance apart in air vacuum to the force of interaction between the same two charges held at the same distance apart in the material medium.

It can also be expressed as

$$K = \varepsilon_r = \frac{\text{Capacitance with dielectric between the plates}}{\text{Capacitance with vacuum between the plates}}$$

$$K = \varepsilon_r = \frac{C_m}{C_o} \qquad ...(15.28)$$

Thus,

$$C_m = KC_o = \varepsilon_r C_o$$

Note : For metal, $K = \infty$ and for mica $K \simeq 6$, paper $K \simeq 3.6$

INTEXT QUESTIONS 15.2

1. *If the potential difference across the plates of a parallel plate capacitor is doubled. what will be change in the capacitance ?*

   ....

2. *Write the dimensions of capacitance 'C'. Give the value of capacitance of parallel plate capacitor in terms of area of plates 'A' and their mutual distance of separation 'd'. Let plates are separated by a dielectric of relative permittivity ε_r*

   ....

3. *Calculate the capacitance of a spherical conductor of radius 10 cm.*

   ....

15.5 GROUPING OF CAPACITORS

To obtain the desired value of capacitance in many electric circuits, capacitors are to be grouped suitably. Two most common modes of grouping of parallel plate capacitors are (i) series grouping, and (ii) parallel grouping.

15.5.1 Series Grouping of Capacitors

In the series combination of capacitors, the first plate of first capacitor is joined to the electric source. The second plate of this capacitor or is joined to the first plate of the next capacitor. The second plate of this capacitor is again connected to first plate of next capacitor of the combination and so on. The second plate of last capacitor of the combination is connected to the earth as shown in Fig 15.9. Let +q unit of charge be given to the first plate of capacitor C_1 from the source. Due to electrical induction, as explained in the principle of capacitor, $-q$ charge appear on the inner side of right plate of C_1 and $+q$ charge develops on the outer side of the second plate of C_1. The $+q$ unit of charge flow to the first plate of C_2 and so on as indicated in Fig. 15.9. Thus, each capacitor receives the same charge of magnitude q. As their capacitances are different, therefore, potential difference across the three capacitors are

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3} \text{, and so on} \quad ...(15.29)$$

If C_s is the total capacitance of the series grouping then,

$$V = \frac{q}{C_s}$$

As, $V = V_1 + V_2 + V_3 + ...$ ...(15.30)

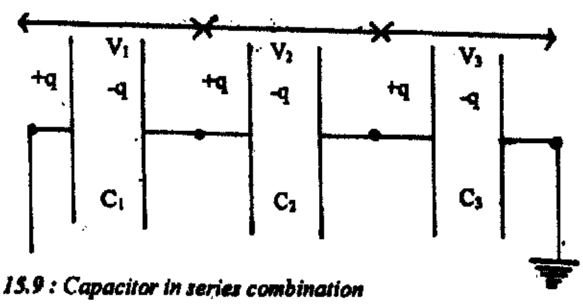


Fig. 15.9 : Capacitor in series combination

$$\therefore \quad \frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} + ...$$

or
$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} .... = \sum_{i=1}^{n} \frac{1}{C_i} \quad ...(15.31)$$

Thus, the ***reciprocal of equivalent capacitance of any number of capacitors joined in series is equal to the sum of the reciprocals of individual capacitances.*** From the above relation, it must the clear to you that C_s is less than the least of C_1, C_2, C_3.......C_n

> **All the capacitors in series grouping have the same amount of charge but the potential difference between their plates are inversely propertional to their capacitances.**

Series combination is used when the high voltage is to be divided on several capacitors. Remember that the lowest capacitor of the combination will have maximum potential difference between its plates.

15.5.2 Parallel Grouping of Capacitors

In this type of grouping, first plates of each capacitor of grouping are connected to one point A and all the second plates to another point B as shown in Fig 15.10. Point B is connected to the earth.

Let V be the potential difference applied to the combination between point A and B. *In this type of combination, unlike the series combination, potential difference across each capacitor is the same*, therefore, charge on them will be different, say q_1, q_2, q_3 ...., such that

$$q_1 = C_1V$$
$$q_2 = C_2V \qquad ...(15.32)$$
$$q_3 = C_3V$$

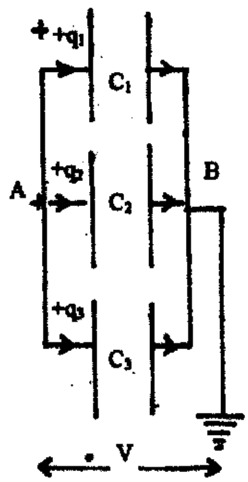


Fig. 15.10: Capacitor in parallel combination

Total charge on all the capacitors of combination is :

$$q = q_1 + q_2 + q_3 + ... \qquad .....(15.33)$$

$$q = (C_1 + C_2 + C_3 + ...)V$$

Let C_p be the equivalent capacitance, in parallel combination, then

$$q = C_pV \qquad ....(15.34)$$

From these relations, we get

$$q = C_pV = (C_1 + C_2 + C_3 + ...)\ V$$

or $$\boxed{C_p = C_1 + C_2 + C_3 + ...} = \sum_{i=1}^{n} C_i$$

Thus, we see that equivalent capacitance of any number of capacitors, joined in parallel is equal to the sum of the individual capacitances.

> Remember in parallel combination all the capacitors have same potential difference between their plates but charge is distributed in proportion to their capacitances. Such combination is used for charge accumulations.

15.5.3 Energy Stored in a Capacitor

The charging of a capacitor can be visualized by imagining as if some external agent, say battery, pulls electrons from the positive plate of a capacitor and transfer them to negative plate. Some work is done in transferring this charge, which is stored in the capacitor, in the form of electrostatic field or potential energy. This energy is obtained from the battery (stored chamical energy). When this capacitor is discharged through a resistance, this energy is released back (in the form of heat etc.). Net electrostatic potential energy of charge capacitor is given as:

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}qV \text{ joule} \qquad ....(15.35)$$

Where all terms carry their usual meaning. This energy of the charged capacitor remains in the dielectric medium between the plates.

15.5.4 Dielectrics and Dielectric Polarization

We know dielectrics are insulating materials which transmit electric effects without conducting. Dielectrics are of two types : non-polar and polar.

(a) Non-polar dielectrics

In the molecules of such type of dielectrics, the centre of positive charge coincides with the centre of negative charge. Each molecule has zero dipole moment in its normal state. These molecules are mostly symmetrical such as nitrogen, oxygen, benzene, methane etc.

(b) Polar dielectrics

These type of dielectrics have asymmetric shape of the molecules such as water, CO_2, NH_3 ,HCl etc. In such molecules, the centres of positive and negative charges do not coincide and they have some permanent dipole moment.

When the non-polar dielectric is held in an external electric field E, the centre of positive charge in each molecule is pushed in the direction of E and centre of negative charge is displaced in the direction opposite to E. Because of external electric field, centres of positive and negative charges in the non-polar dielectric molecules are separated. Dielectric is said to be polarized and a tiny dipole moment is induced in each molecule. In fact, the force due to external electric field pulling the charge centres apart balances with the force of mutual attraction between the centres (i.e. equilibrium is set) and mulecule is said to be polarised.Induced dipole moment p acquired by the molecule may be written as.

$$p = \propto \varepsilon_o E$$

Where $\propto$ is a constant of proportionality and is called atomic/molecular *polarizability*. Let us now consider a non-polar slab ABCD placed in an electric field E maintained between two plates of capaciter. As shown in Fig 15.11 , dielectric slab gets polarised. The nucleus of dielectric molecules are displaced towards the negative plate and electrons towards positive plates. Because of polarisation, an electric field E_p is produced within the dielectric, which is opposite to E. Hence due to the presence of the non-polar dielectric, the field between the plates is reduced. i.e. effective electric field in a polarised dielectric is,

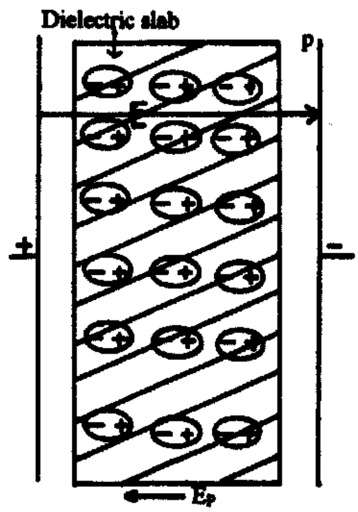


Fig. 5.11: Dielectric polarization between the plates of capacitor

$$E' = E - E_p$$

Thus, the potential difference between the capacitor plates is correspondingly reduced (as $V = E'd$,), increasing the value of capacitance of capacitor (as, $C = q/V$).

INTEXT QUESTIONS 15.3

1. *If air, paper and oil are used as dielectrics between the plates of a capacitor in which case the capacitance will be highest ?*

   ....

2. *Three capacitors having capacitanic 1 μF, 5 μF and 7 μF are grouped (a) in series and (b) in parallel. Capare the equivalent capacitance in both the cases ?*

   ....

3. *A capacitor is charged, removed from the charging battery and a dielectric inserted in between the plates. What will happen to the electrostatic potential energy ? Will it increase, decrease or remain the same?*

   ....

15.6 WHAT YOU HAVE LEARNT

- Electrostatic field is a conservative field.
- The electric potential an any point in at electric field is equal to the work done against the electric force in moving a unit charge from outside the electric field to the given point in electric field.
- Work done in transferring a charge, from one point to another, in an electrostatic field is path independent.
- The capacitance of a conductor increase when another earthed conductor is brought in its vicinity.
- If one joule of work is done in bringing a test charge of one coulomb from infinity to a point in the field, then potential at that point is one volt.
- Electric potential due to a dipole is zero at every point on the equitorial line of dipole.
- Equipotential surface is the surface at every point of which electric potential is same.
- The work done is assembling two charges infinitely apart from each other, is said to be the electrostatic potential energy of the system.
- At any point in an electric field, intensity is equal to negative rate of change of potential with distance (called potential gradient).
- Electrostatic shielding is the phenomenon of protecting a certain region of space from electric field.
- Capacitance of a conductor does not depend upon the material of conductor but depends upon its shape, size and nature of medium.
- The capacitance of a dielectric *filled* paralled plate capacitor becomes K times the capacitance with air or vacuum as dielectric.
- Relative permittivity is the ratio of capacitance with dielectric between the plates to the capacitance with vacuum between the plates.
- In series combination of capacitors, the equivalent capacitance is less than the least of any of the individual capacitance.
- In parallel combination of capacitors, the equivalent capacitance is equal to the sum of individual capacitances.
- Due to the presence of a non-polar dielectric, the field between the plates of a capacitor is reduced.

15.7 TERMINAL QUESTIONS

1. Define electric potential. Is it a scalar or a vector quantity?
2. What is meant by "potential gradient"? Will it have the same unit as potential ?
3. A metallic box is placed in the space having electric field. What is the field inside it ? Explain your answer.

4. Derive the value of potential at any point on the equatorial line of an electric dipole, define the term volt.
5. What is the workdone in moving a charge of 10 C between two points seperated by a distance of 6 cm. on the equipotential surface of 100 μV ?
6. What is a capacitor? Briefly explain the principle of a capacitor. Define the SI Unit of capacitance.
7. What is dielectric ? When a dielectric is inserted in between the plates of a capacitor, how does the capacitance change and why?
8. Derive an expression for the equivalent capacitance of two capacitors C_1 and C_2 connected (a) in series (b) in parallel. Write two application of capacitors in electric circuits.
9. The electric potential is + 80 V at a point distant 80 mm from a point charge. What is the magnitude of charge?
10. In a hydrogen atom. the electron and proton are bound at a distance of about 0.53 Å. Estimate the potential energy of the system in eV, taking the zero of potential energy at infinite separation of electron from the proton.
11. Two capacitors of equal capacitance when connected in series, have a net capacitance C_1 and when connected in parallel, have a net capacitance C_2 What is the value of C_1 C_2?
12. A capacitor C_1 =2μF is charged to a potential of 200V. Another capacitor C_2=4μF is charged to a potential of 400V. Find is charge and energy stored in each capacitor? The two charged capacitors are now joined in parallel. What is their common potential difference ? What is the final energy stored in the system ? Is there a loss of energy ? If yes, what happens to the energy lost?

CHECK YOUR ANSWERS

Intext Questions 15.1

1. Force = charge × electric field intensity
2. Zero, because is rotating the charge in a circle, force is along the radius and direction of motion is perpendicular to it.
3. As $E = \frac{dV}{dx}$ and V is consent. hence $E = 0$
4. No, electric potential will not necessarily be zero, but it can be constant in that region.
5. No. as equipotential surface is always normal to electric field intentiy **E**.

Intext Questions 15.2

1. Capacitance will remain same.
2. $M^{-1}L^{-2}T^4A^2$, $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$
3. 11.1. pF

Intext Questions 15.3

1. oil
2. 611 : 35
3. Potential energy decreases by a factor K, the dielectric constant of the dielectric.

Terminal Questions

10. 0.71 μC
11. 27.17 eV
12. 1/4
13. $Q_1 = 4 \times 10^{-4}$ C, $Q_2 = 16 \times 10^{-4}$C; $E_1 = 4 \times 10^{-2}$ J, $E_2 = 32 \times 10^{-2}$ J,
    The lost energy appears as heat produced in the wire.