

# 2

## MOTION IN A STRAIGHT LINE

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### 2.1 INTRODUCTION

You see a number of things moving around you. People, animals, vehicles can be seen moving on land, fish, frogs and other aquatic animals are seen moving in water. The birds and aeroplane move in air. The sun and the moon appear to move in the sky. Though we do not see but the earth on which we live also moves. It is, therefore, quite apparent that we live in a world that is very much in motion. To understand and describe the physical world around you, the study of motion is very much essential. Motion can be in a straight line, in a plane or in the space. If the motion of the object is in only one direction, it is said to be the "motion in a straight line". For example, motion of a car on a straight road, motion of a train along its straight rails, motion of a freely falling body, motion of a lift, and motion of an athlete running on a straight track.

In this lesson we shall confine our attention to the motion in a straight line leading to the description of motion in general. In the next and the following lessons you will study about the laws of motion, motion in a plane and other various types of motions.

### 2.2 OBJECTIVES

After studying this lesson, you should be able to,

- recall distance, displacement, speed and velocity;
  - explain relative velocity and average velocity;
  - define acceleration and instantaneous acceleration;
  - draw and interpret position - time graph;
  - draw and interpret velocity - time graph for uniform and non-uniform motions;
  - explain instantaneous velocity;
  - derive and apply equations of motion with constant acceleration; and
  - describe motion under gravity.
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## 2.3 VELOCITY AND ACCELERATION

In your earlier classes you would have studied that the total length of the path travelled by a body is **distance** whereas the difference between the initial and final position of the body is called its **displacement**. Basically the displacement is the shortest distance between the two positions and has a certain direction. Thus the displacement is a vector quantity but the distance is a scalar. You would also have learnt that the rate of change of distance with time is called **speed** whereas the rate of change of displacement of a body is known as its **velocity**. Unlike speed, velocity is a vector quantity.

### 2.3.1 Average Velocity

When an object travels with different velocities, its rate of motion is measured by its average velocity or average speed. The average velocity of an object is defined as the rate of change of displacement, whereas the average speed of an object is obtained by dividing the total distance travelled by the total time taken. Let  $x_1$  and  $x_2$  be the positions of the object at  $t_1$  and  $t_2$  times respectively.

Hence, average velocity

$$v = \frac{\text{displacement}}{\text{time taken}}$$

$$\text{i.e. } v = \frac{x_2 - x_1}{t_2 - t_1} \text{ or } v = \frac{\Delta x}{\Delta t} \quad (2.1)$$

$$\text{and, average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

For, understanding the average speed and average velocity clearly, read the following examples.

**Example 2.1 :** An object is moving along the  $x$ -axis whose coordinate is  $x = 20t^2 \text{ ms}^{-2}$ , where  $t$  is the time variable. Calculate the average velocity of the object over the time interval from 3 s to 3.2 s.

**Solution :** Given,

$$x = 20 t^2 \text{ ms}^{-2}$$

We know, the average velocity is given by the relation

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{As } t_1 = 3 \text{ s}$$

$$x_1 = 20 \times (3 \text{ s})^2 \text{ ms}^{-2}$$

$$= \frac{20 \text{ m} \times 9 \text{ s}^2}{\text{s}^2} = 180 \text{ m}$$

$$\text{As } t_2 = 3.2 \text{ s}$$

$$x_2 = 20 \times (3.2 \text{ s})^2 \text{ ms}^{-2}$$

$$= 20 \times 10.2 \text{ m} = 204 \text{ m}$$

$$\therefore v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{204 - 180}{3.2 - 3} = \frac{24}{0.2} \text{ ms}^{-1} = 120 \text{ ms}^{-1}$$

Hence, average velocity =  $120 \text{ ms}^{-1}$

**Example 2.2 :** A man runs on a 300m circular track and comes back on the starting point in 200s. What is the average speed and average velocity of the man?

**Solution :** Given,

Total length of the track = 300 m.

Time taken to cover this length = 200 s

The man comes back to the same point

∴ The displacement = 0

Hence, average speed =  $\frac{\text{Total distance travelled}}{\text{Time taken}}$

$$= \frac{300}{200} \text{ ms}^{-1} = 1.5 \text{ ms}^{-1}$$

and, average velocity =  $\frac{\text{displacement}}{\text{Time taken}}$

$$= \frac{0}{200} = 0 \text{ ms}^{-1}$$

### 2.3.2 Relative Velocity

When we say that a bullock cart is moving at  $10 \text{ km h}^{-1}$  due south, it means the velocity of the cart with respect to the earth is  $10 \text{ km h}^{-1}$ . Infact, the velocity of a body is always specified with respect to some other body. Thus, the *velocity is relative in nature*.

Suppose a girl is walking in the compartment of a moving train in the direction of the motion of the train. It means the girl is moving with respect to the train. Let the train is going due west relative to the surface of the earth. The earth in turn is moving due east relative to the sun. The sun itself is moving around our galaxy. Our galaxy is moving relative to other galaxies. Thus if you want to find the absolute velocity of the girl, it may be a tedious job. All velocities are thus relative.

The relative velocity of an object A with respect to another object B is the rate at which A changes its position relative to B. For example, if  $v_A$  and  $v_B$  are the velocities of the two point objects along a straight line, the relative velocity of B with respect to A will be  $v_B - v_A$ .

The rate of change of the relative position of an object with respect to the other object is known as the **relative velocity** of that object with respect to the other.

**Example 2.3 :** A car A is moving on road from North to South with a speed of  $60 \text{ km/h}$ . Another car B is moving from South to North on the same road with a speed of  $70 \text{ km/h}$ . What is the velocity of car B relative to the car A?

**Solution :** Considering the direction from South to North as positive,

The velocity ( $v_B$ ) of car B =  $+ 70 \text{ km/h}$

And, the velocity ( $v_A$ ) of car A =  $- 60 \text{ km/h}$

Hence, the velocity of car B relative to car A

$$= v_B - v_A \\ = 70 - (-60) = 130 \text{ km/h.}$$

### 2.3.3 Acceleration

While travelling in bus or car, you would have noticed that some time it speeds up and some time slows down, thus changing its velocity. This change occurs with time. Just as the velocity is defined as the time rate of change of position, **the acceleration can be defined as time rate of change of velocity.** The average acceleration of an object is given by,

$$\text{Average acceleration } (a_{av}) = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken for change in velocity}}$$

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad (2.2)$$

The **instantaneous acceleration**  $a$  is defined as the limiting value of the average acceleration, as we let  $\Delta t$  approach zero;

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad (2.3)$$

Acceleration is a vector quantity and its SI unit is  $\text{m/s}^2$ .

Acceleration is a vector and therefore has a direction. In general, for horizontal motion, when the acceleration is in the same direction as the motion or velocity (normally taken to be in the positive direction), the direction is positive. However, an acceleration may be in the opposite direction of the motion. Then the acceleration is taken as negative which is often called a deceleration or **retardation**.

**Example 2.4 :** A cyclist starting from rest attains a velocity of 15 km/h in 3 min, compute the acceleration of the cyclist.

**Solution :** Given,

$$v_1 = 0$$

$$v_2 = 15 \text{ km/h} = \left(\frac{25}{6}\right) \text{ m/s}$$

$$t = 3 \text{ min} = 180 \text{ s.}$$

$$\therefore \text{Acceleration } a = \frac{v_2 - v_1}{t} = \frac{25}{6} \times \frac{1}{180}$$

$$a = 0.023 \text{ m/s}^2$$

Hence, acceleration of cyclist =  $0.023 \text{ m/s}^2$ .

Now, it is time to check your progress. Solve the following questions.

### INTEXT QUESTIONS 2.1

1. Is it possible to have some average speed but average velocity to be zero for a moving object?  
.....
2. A woman went to the market at a speed of 8 km/h. Finding market closed she returned back to her home with a speed of 10 km/h. If the market is 2 km away from her home calculate her average velocity and average speed.  
.....

3. Can a moving body have relative velocity zero with respect to another body? Give an example.  
.....
4. Two cars A and B starts from the same point to move in perpendicular directions with uniform speeds of 30 km/h and 40 km/h respectively. Calculate the relative speed of A with respect to B.  
.....

## 2.4 POSITION - TIME GRAPH

If you roll a ball on ground, you will notice that at different times, the ball has different positions. The different positions and corresponding time can be plotted on graph giving us certain curve. Such a curve is known as position - time curve. Generally, the time is represented on x-axis whereas the position of the body is represented on y-axis.

Let us plot a position - time graph for a body at rest. You see the stationary body does not change its position with time. Hence, *the position-time graph for a stationary body comes to be a straight line parallel to the time axis.* If the body was at a position of 20 m at a certain time and it is at rest, the graph comes to be as shown in Fig 2.1.

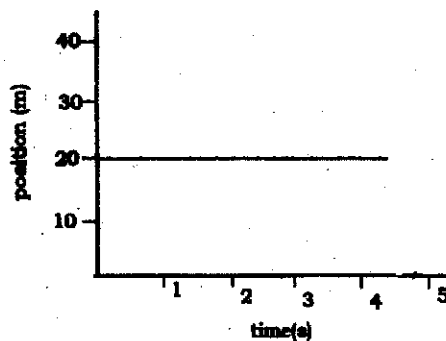


Fig 2.1: Position-time graph for a body at rest.

### 2.4.1 Position-Time Graph for Uniform Motion

Now, let us consider a case when an object covers equal distance in equal interval of time. For example, if the object covers a distance of 10 m in each second for 5 seconds, the positions of the object at different times will as shown in the following table.

Time (t) in (s)	1	2	3	4	5
Position (x) in (m)	10	20	30	40	50

In order to plot this data, take time on x-axis assuming 1 cm as 1 s, and position on y-axis with a scale of 1 cm to be equal to 10 m. The position-time graph will be as shown in Fig 2.2.

*The curve comes to be straight line and it shows that the slope of the curve is constant i.e. the time rate of change of position of the object is constant. It means the speed of the object is uniform. Hence, such a motion in which*

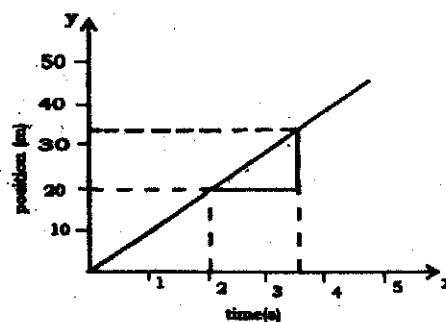


Fig 2.2 : Position-time graph for uniform motion.

**the velocity of the moving object is constant is known as uniform motion.**

In other words we can say that when the moving object covers equal distances in equal intervals of time, it is called **uniform motion**.

For uniform motion the position - time graph is a straight line inclined to the time axis.

### 2.4.2 Position-Time Graph for Non-Uniform Motion

Let us take an example of a train which starts from one station speeds up, then moves with uniform velocity for certain duration and before stopping at other station slows down. In this case you will find that the **distances covered in equal intervals of time are not equal**. Such a motion can be called as **non-uniform motion**. If the distances covered in successive intervals are increasing, the motion is said to be accelerated motion. The position-time graph for such an object is as shown in Fig. 2.3.

From this graph you will notice that the OA, BC and CD portions represent uniform motions but with different velocities. The portion AB shows the stationary position of the body. However, the whole journey shown by the graph represents non-uniform motion.

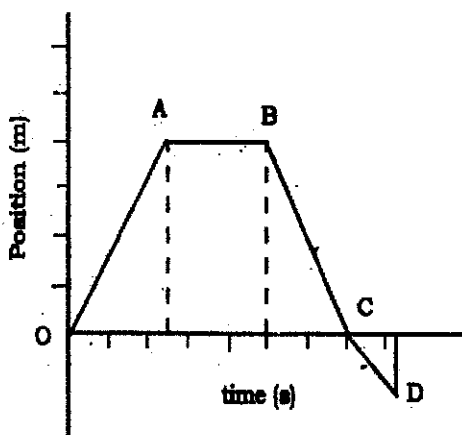


Fig 2.3 : Position-time graph for a non-uniform motion.

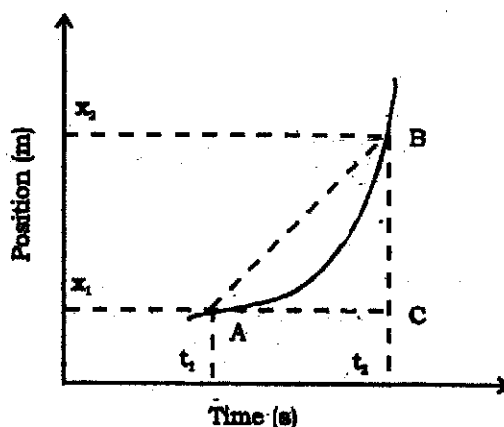


Fig 2.4 : Position-time graph as a continuous curve.

See the Fig. 2.4, the position-time graph can be a continuous curve also. It means the distances covered in different intervals of time are different. Hence, the velocity of the body is changing continuously. In such a situation in any interval of time the average speed of the body can be determined. The **instantaneous speed** can also be determined which will be equal to the slope of the curve at that instant.

### 2.4.3 Interpretation of Position - Time Graph

As you have seen, the position - time graph of different moving objects can be different. If it is straight line parallel to the time axis; you can say that the body is at rest. (Fig 2.3) But the straight line having inclination with time axis shows that the motion is uniform.

**(a) Velocity from position - time graph :** The slope of the straight line of position - time graph gives the velocity of the object in motion. For determining the slope, choose any two points (say A and B) on the straight line (Fig. 2.3) and form a triangle by drawing lines parallel to y axis and x-axis. Thus, from Fig. 2.3, the velocity of the object,

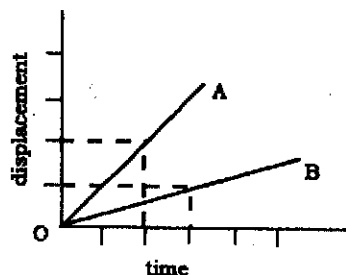
$$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{BC}{AC}$$

Hence, velocity of object = slope of line AB.

It shows that more the slope ( $\Delta x/\Delta t$ ) of the straight line of position - time graph, more will be the velocity. Notice that the slope is also equal to the tangent of the angle that the straight line makes with a horizontal line, i.e.,  $\tan \theta = \Delta x/\Delta t$ . Any two corresponding  $\Delta x$  and  $\Delta t$  intervals can be used to determine the slope and thus the velocity.

**Example 2.5 :** The position - time graph of two bodies A and B is as shown in the figure. Which of them has larger velocity?

**Solution :** The body A has larger velocity. Because the slope of the  $x - t$  graph for body A is greater.



**(b) Instantaneous velocity :** As you have learnt that the velocity of the uniform motion in a straight line is same at every instant. But in the case of the non-uniform motion the position - time graph comes to be a curved line as shown in Fig.2.4. As a result the slope or the average velocity varies, depending on the size of the time intervals selected. In such a case the velocity of the particle at some one instant of time or at some one point of its path, is called its **instantaneous velocity**.

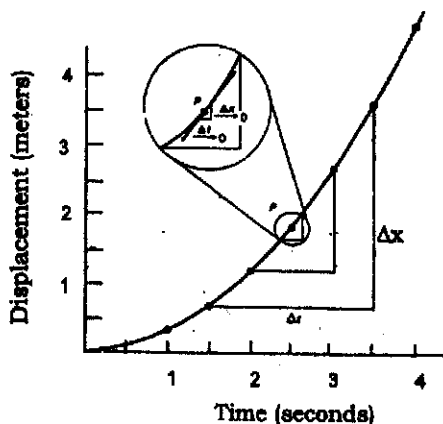
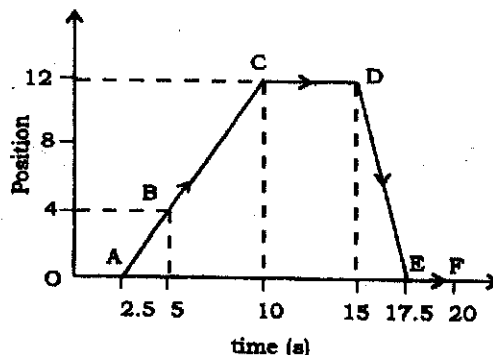


Fig 2.5 : Displacement-time graph for a non-uniform motion.

Taking the limit  $\Delta t \rightarrow 0$ , the slope ( $\Delta x/\Delta t$ ) of a line tangent to the curve at that point gives the instantaneous velocity. However, for uniform motion the average and instantaneous velocities are the same.

**Example 2.6 :** The position - time graph for the motion of a point object is as shown in figure. What distances with what speeds are travelled by object in time (i) 0 s to 5 s, (ii) 5 s to 10 s, (iii) 10 s to 15 s (iv) 15 s to 20 s? What is the average speed for this total journey in time 20 s?



**Solution :**

i) During 0 s to 5 s distance travelled = 4 m

$$\text{The speed} = \frac{\text{Distance}}{\text{Time}} = \frac{4}{5-0} = \frac{4}{5} = 0.8 \text{ m/s}$$

ii) During 5 to 10 s distance travelled = 12 - 4 = 8 m

$$\text{The speed} = \frac{12-4}{10-5} = \frac{8}{5} = 1.6 \text{ m/s}$$

iii) During 10 to 15 s distance travelled = 12 - 12 = 0

The speed = 0

iv) During 15 s to 20 s distance travelled = 12 m

$$\text{The speed} = \frac{\text{Distance}}{\text{Time}} = \frac{12}{20-15} = \frac{12}{5} = 2.4 \text{ m/s}$$

Stop, and solve the following questions to check your progress.

**INTEXT QUESTIONS 2.2**

1. Draw the position - time graph for a motion with zero acceleration.

2. The following figure shows the displacement - time graph for two students A and B who starts from their school and reaches their homes. See it carefully and answer the following.

(i) Who reaches home first?

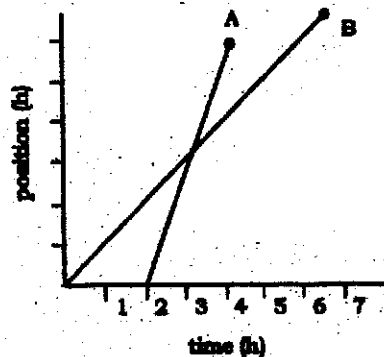
(ii) Who covers more distance?

(iii) Whose speed is less?

(iv) Who moves fast?

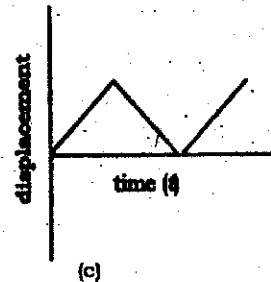
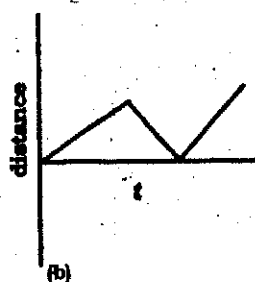
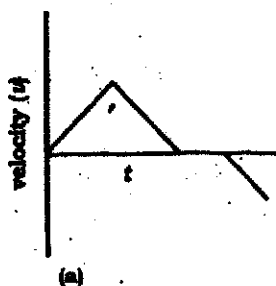
(v) What are the speeds of A and B when they cross each other?

(vi) Who takes minimum and maximum time to reach home?



3. Under what conditions is average velocity equal to instantaneous velocity?

4. Which of the following graph is not possible? Give reason of your answer?



## 2.5 VELOCITY - TIME GRAPH

Just like the position - time graph, we can plot a graph between the velocities of the moving body and the corresponding time. Such a graph can be termed as velocity - time graph. For plotting a velocity - time graph, generally the time is taken on x-axis and the velocity on y-axis.

### 2.5.1 Velocity - Time Graph for Uniform Motion

As you know, in the uniform motion in a straight line the velocity of the body remain constant i.e. there is no change in the velocity with the change in time. The velocity-time graph for such a uniform motion is a straight line parallel to the time axis, as shown in the figure 2.6.

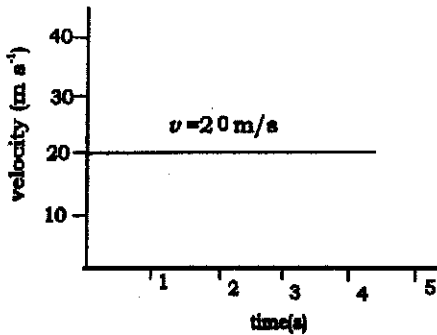


Fig. 2.6 : Velocity-time graph for uniform motion

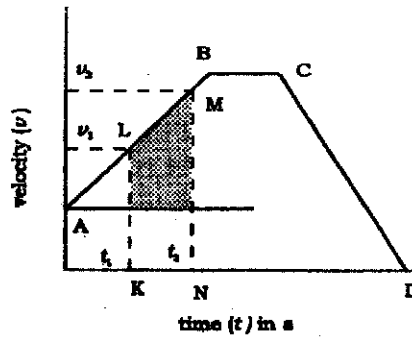


Fig. 2.7: Velocity - Time graph for a motion with constant acceleration.

### 2.5.2 Velocity-Time Graph for Non-Uniform Motion

If the velocity of a body changes uniformly with time, the acceleration is constant. For constant acceleration the average and the instantaneous accelerations are equal.

The velocity - time graph for such a motion of a body, is a straight line inclined to the time axis, as shown in the figure 2.7 by the straight line AB. It is clear from the graph that the velocity increases by equal amounts in equal intervals of time.

Since, the slope of the straight line is constant, the acceleration of the body is constant and its magnitude is given by

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

However, there may be case of non-uniform motion in which the rate of variation in the velocity is not constant. In such a situation, the slope of the velocity - time graph will vary at every instant, as shown in Fig 2.8.

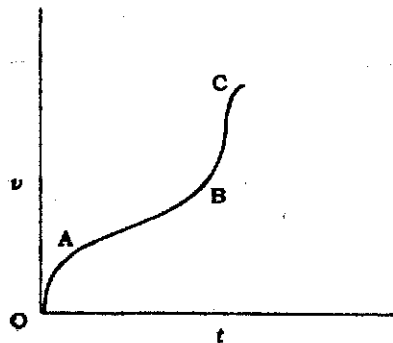


Fig : 2.8: Velocity-time graph for a motion with varying acceleration.



**Solution :**

(i) Since the slope of the  $v-t$  graph gives the acceleration.

As the slope of the  $v-t$  graph for A body is maximum

$\therefore$  its acceleration is maximum.

$$\therefore a = \frac{\Delta v}{\Delta t} = \frac{6-0}{3-0} = \frac{6}{3} = 2 \text{ m/s}^2.$$

(ii) The distance travelled by a body is equal to the area of the  $v-t$  graph

$\therefore$  In first 3 s,

$$\begin{aligned} \text{the distance travelled by A} &= \text{Area OAL} \\ &= \frac{1}{2} \times 6 \times 3 = 9 \text{ m.} \end{aligned}$$

$$\begin{aligned} \text{the distance travelled by B} &= \text{Area OBL} \\ &= \frac{1}{2} \times 3 \times 3 = 4.5 \text{ m.} \end{aligned}$$

$$\text{the distance travelled by C} = \frac{1}{2} \times 1 \times 3 = 1.5 \text{ m.}$$

(iii) At the end of the journey, the maximum distance is travelled by the body B,

$$= \frac{1}{2} \times 6 \times 6 = 18 \text{ m.}$$

(iv) At 2 s, the velocity of A = 4 m/s

the velocity of B = 2 m/s

the velocity of C = 0.80 m/s (approx.)

## 2.6 EQUATIONS OF MOTION WITH CONSTANT ACCELERATION

As we have studied earlier, for describing the motion of an object, the physical quantities like distance, velocity and acceleration are used. For the cases of constant acceleration, the velocity acquired and the distance travelled in a given time can be calculated by using one or more of three equations. These equations, generally known as *equations of motion for constant acceleration* or kinematical equations, are easy to use and will find many applications in this course.

In order to derive these equations, let us take initial time  $t_1$  to be zero i.e.  $t_1 = 0$ . We can then assume  $t_2 = t$  be the elapsed time. The initial position ( $x_1$ ) and initial velocity ( $v_1$ ) of an object will now be represented by  $x_0$  and  $v_0$  and at time  $t$  they will be called  $x$  and  $v$  (rather than  $x_2$  and  $v_2$ ). According to the equation 2.1 the average velocity during the time  $t$  will be

$$v = \frac{x - x_0}{t} \quad (2.4)$$

### 2.7.1 First Equation of Motion

The first equation of motion helps in determining the velocity of an object after a certain time when the acceleration is given.

As you know from the definition of acceleration,

$$\text{Acceleration (a)} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$\therefore a = \frac{v - v_0}{t} \quad (2.5)$$

This equation gives

$$\boxed{v = v_0 + at} \quad (2.6)$$

This is known as the first equation of motion.

**Example 2.8 :** If a car starting from rest has an acceleration of  $10 \text{ m/s}^2$ . How fast will it be going after 5 s?

**Solution :** Given,

Initial velocity  $v_0 = 0$   
 Acceleration  $a = 10 \text{ m/s}^2$   
 Time  $t = 5 \text{ s}$

Using first equation of motion

$$v = v_0 + at$$

After, time  $t = 5 \text{ s}$ , the velocity

$$v = 0 + 10 \times 5$$

$$v = 50 \text{ m/s}$$

### 2.6.2 Second Equation of Motion

Second equation of motion is used to calculate the position of an object after a time  $t$  when it is undergoing constant acceleration  $a$ .

From the definition of average velocity, you know

$$v_{av} = \frac{x - x_0}{t}$$

So,  $x = x_0 + v_{av} t$  (2.7)

Since, the velocity increases at a uniform rate, the average velocity,  $v_{av}$ , will be midway between the initial and final velocities;

$$\therefore v_{av} = (v + v_0)/2 \quad (2.8)$$

Combining the equations (2.7) and (2.8), we get,

$$x = x_0 + \left( \frac{v + v_0}{2} \right) t$$

Putting  $v = v_0 + at$ ,

$$x = x_0 + \left( \frac{v_0 + at + v_0}{2} \right) t$$

or.  $\boxed{x = x_0 + v_0 t + \frac{1}{2} at^2}$  (2.9)

This is known as the second equation of motion.

**Example 2.9 :** A car A is travelling on a straight road with a uniform speed of  $60 \text{ km/h}$ . Another car B following it is moving with uniform velocity of  $70 \text{ km/h}$ . When the distance between them is  $2.5 \text{ km}$ , the car B is given a deceleration of  $20 \text{ km/h}^2$ . At what distance and time will the car B catch up with A?

**Solution :** Suppose the car B catches up car A, at a distance  $x$  after  $t$  time. For car A, the distance travelled in  $t$  time =  $x' = 60 \times t$ . For car B, the distance travelled in  $t$  time is given by

$$\begin{aligned}x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\ &= 0 + 70 \times t + \frac{1}{2} (-20) \times t^2 \\ x &= 70t - 10t^2\end{aligned}$$

But, the distance between two cars is

$$\begin{aligned}x - x' &= 2.5 \\ \therefore (70t - 10t^2) - (60t) &= 2.5 \\ \text{or } 10t^2 - 10t + 2.5 &= 0\end{aligned}$$

It gives  $t = \frac{1}{2}$  hour

$$\begin{aligned}\therefore x &= 70t - 10t^2 \\ &= 70 \times \frac{1}{2} - 10 \times \left(\frac{1}{2}\right)^2 \\ &= 35 - 2.5 = \mathbf{32.5 \text{ km.}}\end{aligned}$$

### 2.6.3 Third Equation of Motion

The third equation is used in a situation when the acceleration, position and initial velocity are known, and the final velocity is desired but the time  $t$  is not known.

From equation (2.7), you have.

$$\begin{aligned}x &= x_0 + v_{av} \cdot t \\ \text{or } x &= x_0 + \left(\frac{v + v_0}{2}\right) t\end{aligned}$$

But from equation (2.5), you have

$$t = \frac{v - v_0}{a}$$

Substituting this into the above equation, we get

$$\begin{aligned}x &= x_0 + \left(\frac{v + v_0}{2}\right) \left(\frac{v - v_0}{a}\right) \\ x &= x_0 + \left(\frac{v^2 - v_0^2}{2a}\right)\end{aligned}$$

On solving this for  $v^2$ , we obtain,

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)} \quad (2.10)$$

This is known as third equation of motion.

Thus, the three equations for constant acceleration are,

$$\boxed{\begin{aligned}v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\ \text{and } v^2 &= v_0^2 + 2a(x - x_0)\end{aligned}}$$

**Example 2.10 :** A motorcyclist moves along a straight road with a constant acceleration  $4 \text{ ms}^{-2}$ . If initially she was at a position of  $5 \text{ m}$  and had a velocity of  $3 \text{ ms}^{-1}$ , find

- the position and velocity at time  $t = 2 \text{ s}$ .
- the position of the motorcyclist when its velocity is  $5 \text{ ms}^{-1}$ .

**Solution :** We are given,

$$x_0 = 5 \text{ m}, v_0 = 3 \text{ ms}^{-1}, a = 4 \text{ ms}^{-2}.$$

- Using equation

$$\begin{aligned}
 x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\
 &= 5 + 3 \times 2 + \frac{1}{2} \times 4 \times (2)^2
 \end{aligned}$$

Position,  $x = 19 \text{ m}$ .

From equation

$$\begin{aligned}
 v &= v_0 + at \\
 &= 3 + 4 \times 2
 \end{aligned}$$

Velocity,  $v = 11 \text{ ms}^{-2}$ .

(ii) Using equation

$$\begin{aligned}
 v^2 &= v_0^2 + 2a(x - x_0) \\
 (11)^2 &= (3)^2 + 2 \times 4 \times (x - 5)
 \end{aligned}$$

We get,  $x = 7 \text{ m}$ .

Hence position of the motorcyclist ( $x$ ) = 7 m.

## 2.6.4 Motion under gravity

You must have noticed that when we throw a body in the upward direction or drop a stone from a certain height, in both the cases they come down to the earth. Do you know why they come to the earth and what type of path they follow? It is because of the gravitational force acting on them. Such type of motions under the influence of gravity only, are also one dimensional or motion in a straight line. **The free fall of a body towards the earth is one of the most common example of motion with (nearly) constant acceleration.** In the absence of air resistance it is found that all bodies, irrespective of their size or weight, fall with the same acceleration at the same point on the earth's surface. Though the acceleration due to gravity varies with altitude, but for the small distances compared to the earth's radius, it remains constant throughout the fall. For our practical use the effect of air resistance and the variation in acceleration with altitude is taken negligible.

The acceleration of a freely falling body due to gravity is denoted by " $g$ ". At or near the earth's surface its magnitude is approximately  $9.8 \text{ m/s}^2$ . More precise values, and its variation with height and latitude will be discussed in detail in the lesson 5 of this booklet.

**Example 2.11** : A stone is dropped from a height of 10 m and it falls freely. Calculate the following,

- (i) distance travelled in 2 s.
- (ii) velocity of the stone when it reaches the ground.
- (iii) velocity at 3 s.

**Solution** : Given,

Height ( $h$ ) = 10 m.

Initial velocity ( $v_0$ ) = 0

Considering, initial position ( $y_0$ ) to be zero and the origin 0 at the starting point. Thus, the  $y$  - axis (vertical axis) below it will be negative. Since, the acceleration is downward in the negative  $y$  - direction, the value of  $a = -g = -9.8 \text{ m/s}^2$ .

(i) Using the equation (2.9),

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

We get,

$$\begin{aligned}
 y &= 0 + 0 - \frac{1}{2} g t^2 = -\frac{1}{2} \times 9.8 \times (2)^2 \\
 &= -19.6 \text{ m.}
 \end{aligned}$$

The negative sign shows that the distance is below the starting point in downward direction.

(ii) At the ground  $y = -10$  m,

Using equation (2.10),

$$v^2 = v_0^2 + 2 a (y - y_0)$$

$$= 0 + 2 (-9.8) (-10 - 0)$$

$$v = 14 \text{ m/s.}$$

(iii) Using  $v = v_0 + at$

$$\text{at } t = 3 \text{ s}$$

$$v = -29.4 \text{ m/s}$$

This shows that the velocity of the stone at  $t = 3$  s is 29.4 m/s and it is in downward direction.

Take a pause and solve the following questions.

**Note :** It is important to mention here that in the above example sign of the acceleration, velocity and position will be positive if you take origin at the ground level.

### INTEXT QUESTIONS 2.3

1. A body starting from rest covers a distance of 40 m in 4 s with constant acceleration along a straight line. Compute its final velocity and the time required to cover half of the total distance.  
.....
2. A car moves along a straight road with constant acceleration  $5 \text{ m/s}^2$ . Initially at 5 m its velocity was 3 m/s. Compute its position and velocity at  $t=2$  s.  
.....
3. With what velocity should a body be thrown vertically upward so that it reaches a height of 25 m? And how long will it be in air?  
.....
4. A ball is thrown upward in the air. Is its acceleration greater while it is being thrown or after it is thrown?  
.....

### 2.7 WHAT YOU HAVE LEARNT

- The ratio of the displacement of an object to the time interval is known as average velocity.
- The total distance travelled divided by the time taken is average speed.
- The rate of change of the relative position of an object with respect to the other object is known as the relative velocity of that object with respect to the other.
- The change in the velocity in unit time is called acceleration.
- The position-time graph for a body at rest, is a straight line parallel to the time axis.
- The position - time graph for a uniform motion is a straight line inclined to the time axis.
- A body covering equal distance in equal intervals of time is said to be in uniform motion.
- The velocity of a particle at some one instant of time or at some one point of its path is called its instantaneous velocity.
- The slope of the position - time graph gives the velocity.
- The velocity - time graph for a body moving with constant acceleration is a straight line inclined to the time axis.

- The area under the velocity - time graph gives the distance travelled.
- The acceleration of the body can be computed by the slope of velocity- time graph.
- For explaining the motion of a body, following three equation are used,
  - (i)  $v = v_0 + a.t$
  - (ii)  $x = x_0 + v_0 t + \frac{1}{2} a t^2$
  - (iii)  $v^2 = v_0^2 + 2 a.(x - x_0)$

## 2.8 TERMINAL QUESTIONS

1. Distinguish between average speed and average velocity.
2. A car A moving with a speed of 65 km/h on a straight road, is ahead of motorcycle B moving with the speed of 80 km/h in the same direction. What is the velocity of B relative to A?
3. How long does a car take to travel 30 m, if it accelerates from rest at a rate of 2.0 m/s<sup>2</sup>?
4. A motorcyclist covers half of the distance between two places at a speed of 30 km/h and the second half at the speed of 60 km/h. Compute the average speed of the motorcycle.
5. A duck flying directly south for the winter flies with a constant velocity of 20 km/h for 25 km. How long does it take for the duck to fly this distance?
6. Bangalore is 1200 km from New Delhi by air (straight line distance) and 1500 km by train. If by air it takes 2 hrs and by train 20 hrs, find the ratio of the average speed in both cases.
7. A car accelerates along a straight road from rest to 50 km/h in 5.0 s. What is the magnitude of its average acceleration?
8. A body with an initial velocity of 2.0 m/s is accelerated at 8.0 m/s<sup>2</sup> for 3 seconds. (i) How far does the body travel during the period of acceleration? (ii) How far would the body travel if it were initially at rest?
9. A ball is released from rest from the top of a cliff. Taking the top of the cliff as the reference (zero) level and upwards as the positive direction. Draw (i) the displacement-time graph, (ii) distance-time graph. (iii) velocity-time graph, (iv) speed time graph.
10. A ball thrown vertically upwards with a velocity  $v_0$  from the top of the cliff of height  $h$ , falls to the beach below. Taking beach as the reference (zero) level, upward as the positive direction, draw the motion graphs. i.e. the graphs between (i) distance - time, (ii) velocity - time, (iii) displacement - time, (iv) speed - time graphs.
11. A body is thrown vertically upward, with a velocity of 10 m/s. What will be the value of the velocity and acceleration of the body at the highest point?
12. Two objects of different masses, one of 10 g and other of 100 g are dropped from the same height. Will they reach the ground at the same time? Explain your answer.
13. What happens to the uniform motion of a body when it is given an acceleration at right angle to its motion?
14. What does the slope of velocity-time graph at any instant represent?

## 2.9 ANSWERS TO THE INTEXT QUESTIONS

### Intext Questions 2.1

1. Yes
2. Average speed =  $\frac{2+2}{\frac{2}{8} + \frac{2}{10}} = \frac{4}{9} \times 20$   
= 8.89 km/h, average velocity = 0
3. Yes, two cars moving with same velocity in the same direction.
4. After 1 hr, the relative distance between A and B will be  
=  $\sqrt{(30)^2 + (40)^2} = 50$  km.

Hence, relative speed of A with respect to B is 50 km/h.

**Intext Questions 2.2**

1. See Fig 2.2
2. (i) A, (ii) B covers more distance, (iii) B, (iv) A, (v) Speed of A = 1.5 km/h, speed of B = 0.75 km/h, (vii) A takes minimum time.
3. In the uniform motion.
4. (a) is not possible, because the distance covered cannot be less or zero.

**Intext Questions 2.3**

1. Using  $x = x_0 + v_0 t + \frac{1}{2} a t^2$   
 $a = 5 \text{ ms}^{-2}$

Next using  $v^2 = v_0^2 + 2a(x - x_0)$

$$v = 10\sqrt{2} \text{ ms}^{-1}$$

2. Using eq (2.9),  $x = 21 \text{ m}$ , and using eq (2.6),  $v = 13 \text{ ms}^{-1}$ .
3. At maximum height  $v = 0$ , using eq (2.10),  $v_0 = 7\sqrt{10} \text{ ms}^{-1} = 22.6 \text{ ms}^{-1}$ .  
 The body will be in air for the twice of the time it takes to reach the maximum height  
 $= 4.5/\text{s}$ .
4. The acceleration of the ball is greater while it is thrown.