20

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

20.1 INTRODUCTION

Now-a-days you can illuminate a dark room instantly by merely turning on a switch. Have you wondered what makes this possible? To find an answer to this question we suggest that you follow the wires connected to the switches. Where do they lead ? To transmission lines outside your homes, at the end of which you will find an ***electric generator*** situated in a Rydro or thermal power plant. At many places between your homes and these generators, there are several substations containing ***transformers.*** Both these devices, the electric generator and transformer are essential for large scale generation and distribution of electrical power today. Both these are based on the phenomenon of ***electromagnetic induction.*** It was discovered independently by Michael Faraday and Joseph Henry more than 250 years ago.

In this lesson we shall discuss electromagnetic induction and the laws governing it, namely, ***Faraday's law*** and ***Lenz's law.*** This phenomenon may, in a sense, be called the electric effect of changing magnetic fields. We shall study some consequences of this phenomenon. In particular, the inductance of coils and a.c. circuits along with some of their applications. In the next lesson you will study about generators and transformers which form the cornerstone of electrical power generation and its transmission.

20.2 OBJECTIVES

After studying this lesson, you should be able to:

- *state and explain Faraday's law of electromagnetic induction;*
- *apply Lenz's law and explain conservation of energy on the basis of Lenz's law;*
- *explain the phenomena of self-inductance and mutual inductance and distinguish between them;*
- *discuss applications of electromagnetic induction;*
- *define alternating current and voltage, their peak and rms values, and represent them graphically;*
- *derive the relationship between voltage and current in a.c. circuits containing resistance; capacitance, inductance only and in an LCR circuit;*
- *derive expression for power dissipated in a.c.circuits and define power factor and discuss applications of a.c. circuits.*

20.3 ELECTROMAGNETIC INDUCTION

Since a steady current in a wire produces a steady magnetic field, Faraday initially (and mistakenly) thought that a steady magnetic field could produce a current. Some of Faraday's investigations on magnetically induced currents used an arrangement similar to that shown in Fig.20.1. A current in the coil on the left produces a magnetic field concentrated in the iron ring. The coil on the right is connected to a galvanometer G, which indicates the presence of any induced current in that circuit. There is no induced current for a steady magnetic field. But an induced current does appear for a moment in the circuit on the right when switch S is closed in the circuit on the left. When switch S is opened, an induced current with the opposite sense appears for a moment. Thus, the induced current exists only when the magnetic field, due to the current in the circuit on the left, is ***changing***.

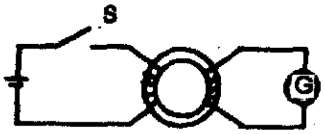


***Fig. 20.1:** Two coils are wrapped around an iron ring. The galvanometer G deflects for a moment when the switch is opened or closed.*

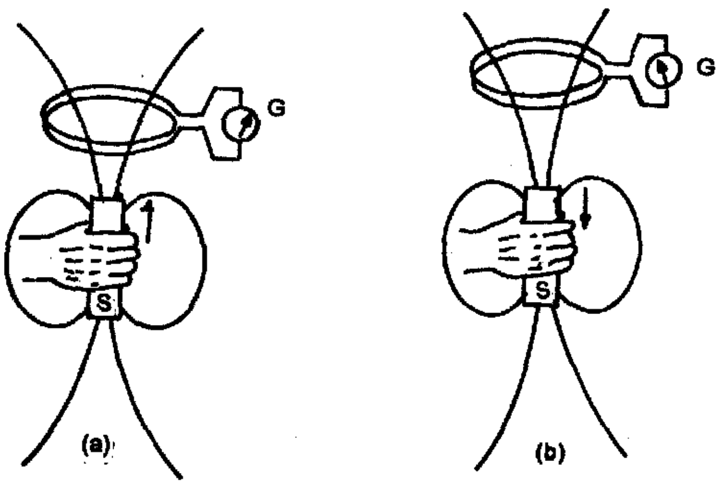


***Fig.20.2 :** (a) A current is induced in the coil if the magnet moves towards the coil; (b) the induced current has the opposite sense if the magnet moves away from the coil.*

The importance of a change is also demonstrated by the arrangement shown in Fig. 20.2. If the magnet is at rest relative to the coil, then no induced current exists. But if the magnet is moved toward the coil, then a current is induced with a sense as indicated in Fig. 20.2a. If the magnet is moved away from the coil, then a current is induced with the opposite sense, as shown in Fig. 20.2b. Notice that in either case the magnetic field is changing in the neighbourhood of the coil. An induced current also exists in the coil if it is moved relative to the magnet.

The presence of such currents in a circuit implies the existence of an ***induced electromotive force (emf)***.

That is, energy must be supplied to the charge carriers that make up the current. You know that an emf is the energy per unit charge given to a charge carrier that travels around the circuit. ***This induced emf is present when the magnetic field is changing,*** as described above.

This phenomenon of a changing magnetic field inducing an emf or an electric field is termed ***electromagnetic induction***. It was a mark of Faraday's genius that he recognised the significance of his discovery and followed it up. He also gave the quantitative description of this phenomenon. We now know it as Faraday's law of electromagnetic induction. Let us learn about it.

20.3.1 Faraday's Law of Electromagnetic Induction

The relationship between the changing magnetic field and the induced emf is expressed in terms of the magnetic flux ϕ_B for a surface. You will now ask: What is magnetic flux?

Let us first define the magnetic flux ϕ_B of the magnetic field for a surface. Imagine dividing a mathematical surface into infinitesimal area elements. The direction of an area element dS at a point on the surface is perpendicular to the surface at that point. A typical element for a surface is shown in Fig. 20.3a, along with the magnetic field **B** at a point. The magnetic flux $d\phi_B$ for the area element dS is defined as

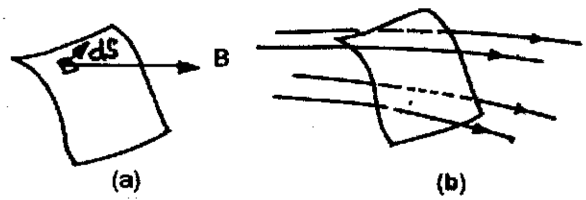


Fig.20.3 (a)The magnetic flux for an infinitesimal area dS is given by $d\phi_B = \mathbf{B}.\,d\mathbf{S}$: (b) The magnetic flux for a surface is proportional to the number of lines intersecting the surface.

$$d\phi_B = \mathbf{B}\,.\,d\mathbf{S} \qquad \text{....(20.1a)}$$

The magnetic flux for a general surface is obtained by integrating (summing) the contributions $d\phi_B$ as the area element dS ranges over the surface. Thus,

$$\phi_B = \quad \mathbf{B}\,.\,d\mathbf{S} \qquad \text{...(20.1b)}$$

*The **magnetic flux** over the surface for a surface is the surface integral of the magnetic field.*

The SI unit of magnetic flux is the ***weber*** (Wb), with 1 Wb = 1 $T.m^2$.

The magnetic flux for a surface can be interpreted in terms of the magnetic lines that represent the distribution of the magnetic field in space. In analogy with electric lines and as suggested in Fig. 20.3b, the number of magnetic lines intersecting a surface is proportional to the magnetic flux for the surface.

Now for simplicity, let us consider a fine loop of conducting wire and an open, mathematical surface bounded by the loop such as the one shown in Fig. 20.4. The magnetic flux for a surface bounded by the loop is given by the surface integral

$$\phi_B = \int \mathbf{B}\,.d\mathbf{S}$$

where $d\phi_B = \mathbf{B} . d\mathbf{S} = B\, dS \cos\theta$ is the flux for the surface element dS. The magnetic flux ϕ_B is said to link the loop.

A changing magnetic flux linking a loop and the induced emf in the loop are related by ***Faraday's law*** :

An emf is induced in a loop of wire when the magnetic flux for a surface bounded by the loop changes in time. The induced emf is given by,

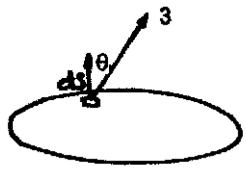


Fig. 20.4: A conducting loop forms the boundary of a surface

$$\varepsilon = -\frac{d\phi_B}{dt} \qquad ....(20.2a)$$

The emf ε depends on the rate of change of the magnetic flux with time. From Faraday's law, we obtain the relation between the weber (Wb), the unit of magnetic flux, and the volt (V) , the unit of emf : 1V=1 Wb/s.

Notice the negative sign in Faraday's law. You will learn about its importance in the next sub-section on Lenz's law. This particular mathematical form of the law (Eq.20.2a) was developed long after Faraday's work, but the law is named after him because he made the key discovery and gave the quantitative relationship.

Now consider the induced emf in a closely wound coil. Each turn in such a coil behaves approximately as a single loop, and we can apply Faraday's law to determine the emf induced in each turn. Since the turns are in series, the total induced emf ε_T in a coil is the sum of the emfs induced in each turn. We suppose that the coil is so closely wound that the magnetic flux linking a turn of the coil at a given instant has the same value for each turn. Then the same emf ε is induced in each turn, and the total induced emf for a coil with N turns is given by

$$\boxed{\varepsilon_T = N\varepsilon = N\left(-\frac{d\phi_B}{dt}\right) = -N\,\frac{d\phi_B}{dt}} \qquad ...(20.2b)$$

where ϕ_B is the magnetic flux linking to a single turn (or loop) in the coil.

The magnetic flux linking a loop or turn of a coil in Eq. (20.2a) is the flux of the total magnetic field for a surface bounded by the loop. There is a contribution to the magnetic flux for a loop due to the loop's own current in addition to the contribution due to an external source, such as a magnet, or the current in another circuit. You will read about it in the next section. In this section, we shall always assume that loops or coils are part of a circuit with a large resistance so that the induced current is small. Then the flux contribution due to the small induced current is assumed to be negligible compared with the flux due to the other sources. Thus, we shall neglect the effect of the induced current in determining the magnitude of the induced emf.

Let us now apply Faraday's law to some concrete situations :

Example 20.1 : *The axis of a 75-turn circular coil of radius 35 mm is parallel to a spatially uniform magnetic field. The magnitude of the field changes at a constant rate from*

25 to 50 mT in 250 ms. Determine the magnitude of induced emf in the coil during this time interval.

Solution : Since the magnetic field is spatially uniform and parallel to the axis of the coil, the flux linking each turn is given by

$$\phi_B = B\pi R^2$$

where R is the radius of a turn. From Eq. (20.2b) the induced emf in the coil is

$$\varepsilon_T = -N\frac{d\phi_B}{dt} = -N\frac{d(B\pi R^2)}{dt} = -N\pi R^2\frac{dB}{dt}$$

The magnitude of the magnetic field changes at a constant rate given by

$$\frac{dB}{dt} = \frac{0.050\ T - 0.025\ T}{0.25\ s} = 0.10\ T/s$$

The magnitude of the emf induced in the coil is then

$$\varepsilon_T = 75\ \pi\ (0.035\ m)^2\ (0.10\ T/s) = 0.030\ V = 30\ mV$$

This example explains the concept of emf induced by a time changing magnetic field.

Example 20.2 : *Consider a long solenoid with a cross-sectional area of 8 cm² (Figs. 20.5a and 20.5b). A time-dependent current in the wire winding creates a time-dependent magnetic field $B(t) = B_o \sin 2\pi\nu t$. Here B_o is constant, and equal to 1.2 T. The quantity ν is the frequency of the magnetic field. With ν = 50 Hz and the ring resistance $R = 1.0\Omega$. Calculate the emf and the current (I) induced in a ring of radius r concentric with the axis of the solenoid.*

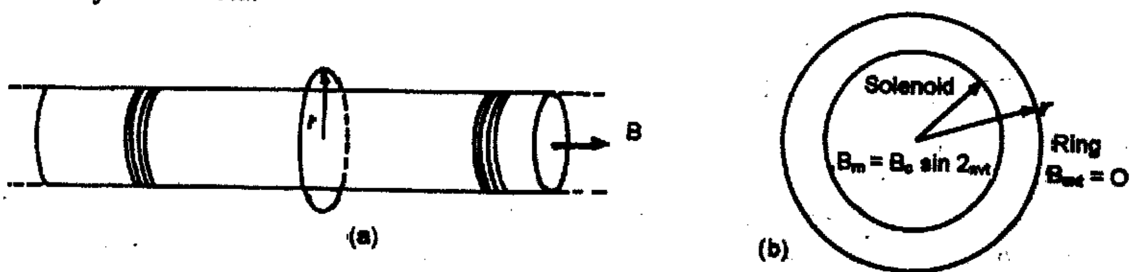


Fig. 20.5 : (a) A long solenoid and a concentric ring outside the solenoid. (b) Solenoid and concentric ring : a cross-sectional view.

Solution : The magnetic flux is equal to

$$\phi_B = B_o \sin 2\pi\nu t\ .\ A$$

and so $$\varepsilon = -\frac{d\Phi_B}{dt} = -2\ \pi\nu A\ B_o \cos 2\pi\nu t$$

$$= -2\pi\ .\ 50\ s^{-1}\ .8 \times 10^{-4}\ m^2 (1.2\ T)\ .\cos 2\pi\nu t$$

$$= -\ 0.250\ \cos 2\pi\nu t\ V$$

The current in the ring is $I = \varepsilon/R$. Therefore,

$$I = \frac{-0.25 \cos 2\pi vt \text{ V}}{1.0}$$
$$= -0.25 \cos 2\pi vt \text{ A}$$

The induced current oscillates with frequency 50 Hz and has an amplitude of 0.25m A. This example explains the conecpt of electromagnetic induction by a time dependent field. You may like to do a few exercises before you study further.

INTEXT QUESTIONS 20.1

1. *A 1000 turn coil has a radius of 5 cm. What emf is developed across the coil if the magnetic field through the coil is reduced from 10 T to 0 in (a) 1 s (b) 1 ms ?*

   ....

2. *The magnetic flux linking each loop of a 250-turn coil is given by the expression $\phi_B(t) = A + Dt^2$, Where $A = 3.0$ mWb and $D = 15$ mWb/s² are constants. (a) Show that the magnitude of the induced emf in the coil is given by $\varepsilon = (2ND)t$. (b) Evaluate the flux linking each turn at t ε= 0.0, 1.0, 2.0, and 3.0 s. (c) Evaluate the induced emf in the coil at each of these instants.*

   ....

3. *The perpendicular to the plane of a conducting loop makes a fixed angle θ with a spatially uniform magnetic field. If the loop has area S and the magnitude of the field changes at a rate dB/dt, show that the magnitude of the induced emf in the loop is given by $\varepsilon = |(dB/dt)\, S \cos\theta|$. For what orientation (s) of the loop is ε, a maximum? and a minimum?*

   ....

20.3.2 Lenz's Law

Let us look again at Faraday's law. Why is the minus sign there ? What are its implications?

To answer these questions consider a bar magnet approaching a conducting ring (Fig. 20.6a). To apply Faraday's law to this system we first choose a positive direction around the ring. Let us take the direction from z to x as positive. (Either choice is fine, as long as we are consistent.) For our choice the positive normal for the area of the ring is in the Y– direction and the magnetic flux is negative. As the distance between the conducting ring and the N–pole of the bar magnet decreases, more and more field lines go through the ring, making the flux more and more negative. Thus $d\phi_B/dt$ is negative. By Faraday's law ε is positive relative to our chosen direction. The current I is directed as shown.

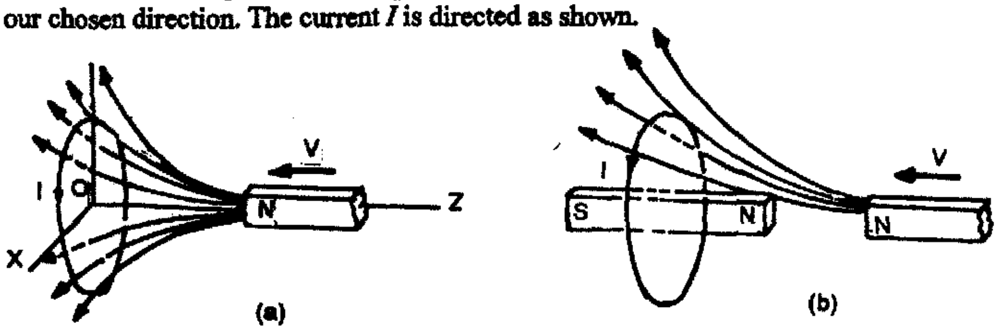


Fig. 20.6: (a) A bar magnet approaching a metal ring; (b) the magnetic field of the induced current opposes the approaching bar magnet.

The current induced in the ring creates an induced secondary magnetic field that is opposite to the original field inside the ring. This induced magnetic field is similar in form to the field of a second bar magnet, shown dotted in Fig. 20.6b. This induced magnetic field repels the incoming magnetic field. This opposition is a consequence of the minus sign in Faraday's law, and is formalized as Lenz's law. ***When a current is induced in a conductor, the direction of the current will be such that the current's magnetic effects oppose the change that induced it.***

The key word in this statement is ***oppose*** – it tells us that we are not going to get something for nothing. When the bar magnet is pushed toward the ring. the current induced in the ring creates a magnetic field that opposes the change in flux. The magnetic field produced by the induced current repels the incoming magnet. If we push the magnet toward the ring, we must do work on the magnet to move it inward. The work we do shows up as electrical energy in the ring. Lenz's law thus follows from the law of conservation of energy.

Suppose that Faraday's law did not have the minus sign. Then the current in the ring would be in the reverse direction. And the magnetic field produced by the induced current would ***reinforce*** the original field. Thus $d\phi_B/dt$ would increase, increasing the induced emf and hence the induced current. The magnetic field of the induced current would attract the incoming magnet. Thus, the magnet would do mechanical work and we would get electric current in the conducting ring. In effect we would have created energy out of nothing violating the law of conservation of energy. However, the world does not work in this way. Lenz's law tells us that if we want to get electrical energy out of this system, we will have to do work. Faraday's law will have a minus sign.

As an application of Lenz's law, consider the coil in Example 20.1.Suppose its axis is vertical and the magnetic field is directed vertically upward. To an observer located directly above the coil, what is the sense of the induced emf? It is clockwise because only then the magnetic field due to it (directed downward by the right-hand rule) will oppose the changing magnetic flux. You should get familiar with Lenz's law before studying further. Try the following exercises.

INTEXT QUESTIONS 20.2

1. *The bar magnet in Fig.20.7 moves to the right. What is the sense of the induced current in the stationary loop A? In loop B?*

   ....

2. *A cross-section of an ideal solenoid is shown in Fig 20.8 . The magnitude of the spatially uniform field is increasing inside the solenoid and B = O outside the solenoid. In which of the conducting loops is there an induced current? What is the sense of each current ?*

   ....

3. *A bar magnet, aligned with its axis along the axis of a copper ring, is moved along its length toward the ring. Is there an induced current in the ring ? Is there an induced electric field in the ring? Is there* a magenetic force on the bar magnet? Explain.

   ....

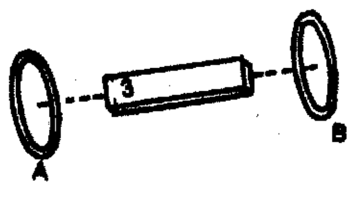


Fig. 20.7:

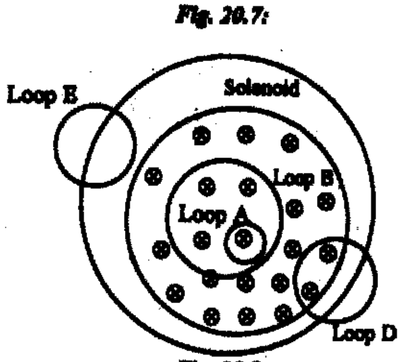


Fig. 20.8

One of the applications of Faraday's Law is to determine the behaviour of simple circuit elements in circuits with currents that change. An induced emf can appear in a circuit element called an ***inductor*** which possesses an ***inductance***. An inductor can store magnetic energy. Recall that the concepts of resistance and capacitance help us to evaluate energy conversion and energy storage in electrical circuits. The concept of inductance helps us to evaluate the amount of magnetic energy sotred by a solenoid. Let us study more about the phenomenon.

20.4 INDUCTANCE

When a current changes in a circuit, a changing magnetic field is produced around it. If a part of this field passes through the circuit itself then an emf is induced in it. Now suppose there is another circuit in the neighbourhood of this circuit. Then the magnetic field through that circuit changes, inducing an emf in it. Thus, induced emfs can appear in circuits in two ways :

i) by changing the magnetic field linked to a coil, an induced emf appears in that coil– this property is called ***self-inductance*** of the coil.

ii) for a pair of coils situated near so that the flux associated with one coil passes through the other, a changing current in one coil induces an emf in the other. In this case we speak of ***mutual– inductance*** of the coil.

20.4.1 Self-Inductance

Let us consider a loop of a conducting material. The loop contains an electric current. The current produces a magetic field, **B**. The magnetic field gives rise to a magnetic flux. The total magnetic flux linking the loop is

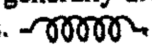
$$d\phi = \mathbf{B} \cdot d\mathbf{S}$$

In the absence of any external source of magnetic flux (for example, an adjacent coil carrying a current), the Biot-Savart's law requires that the magetic field be proportional to the current in the loop. Therefore the flux threading the loop is also proportional to the current (I) in the loop

$$\phi \propto I \qquad \text{....(20.3)}$$

The proportionality constant between magnetic flux and current is called the ***self-inductance*** of the loop. Self-inductance is given the symbol L, and is defined by

$$\boxed{\phi = LI} \qquad \text{....(20.4)}$$

Thus, the current in the loop and the magnetic flux threading the loop are related by the property of ***self-inductance*** of the loop. Now, the loop might be an intricate connection of components in an integrated electronic circuit or it might be a carefully engineered coil of wire. Self-inductance can exist in wires, electrical transmission lines, and arbitrarily shaped loops. For example, self-inductance is a very important characteristic of a coaxial transmission line. However, we are usually interested in circuit elements that are disigned to exploit inductive effects. Such circuit elements are called ***inductors*** and generally are coils of wire of varied shapes and sizes. The symbol for an ***inductor*** is.  If the coil is wrapped around an iron core so as to enhance its magnetic effect. it is symbolized by .

(a) Faraday's Law in Terms of Self-Inductance: So far you have studied that if the current in a loop is changed, the magenetic flux threading the loop changes. This change in magnetic flux is accompained by an induced emf in the loop, called a self-induced emf. At any moment the current in the loop is determined by both the self-induced emf and the potential difference created by the imposed emf (a battery, for example) between the ends of loop. In accordance with Lenz's law, the self-induced emf in this case (closing the switch) opposes the change that produces it.

For self-induced emfs in a circuit, let us now obtain a form of Faraday's law relating the induced emf to the rate of change of current. We begin by differentating Eq. (20.4) with respect to time to obtain

$$= \frac{d\phi}{dt} = L\frac{dI}{dt} \qquad .....(20.5)$$

Using Faraday's law of induction, we can replace $d\Phi/dt$ with the negative of the induced emf $(-\varepsilon)$ so that

$$\boxed{\varepsilon = -L\frac{dI}{dt}} \qquad .....(20.6)$$

According to this equation, the induced emf is proportional to the time rate of change of current in the loop. This is the form of Faraday's law that is applicable to electrical circuits. Note that a constant current, no matter how large, produces zero induced emf, whereas a current changing rapidly produces a large induced emf. The units of self-inductance are-

$$\text{units of } L = \frac{\text{units of}}{\text{units of } dI/dt}$$

$$= \frac{\text{volts}}{\text{amperes/second}}$$

$$= \text{ohm-second}$$

An ohm-second is called a *henry*, abbreviated H. For most applications, the henry is a rather large unit, and we often use the more convenient units millihenry, mH (10^{-3} H), and microhenry μH (10^{-6}H) instead.

This induced emf is also called the ***back emf.*** Eq. (20.6), tells us that the ***back emf in an inductor*** depends on the rate of change of the inductor current and ***acts to oppose that change in current.*** Since an infinite emf is impossible, so from Eq. (20.6), an instantaneous change in the inductor current cannot occur. Thus, we can say that. ***The current through an inductor cannot change instantaneously.***

You have just studied that the self-inductance of an inductor is a measure of the opposition to the change in current through it. How do we determine the magnitude of the self-inductance of an inductor?

The inductance of an inductor depends on its geometry. In principle, we can calculate the self-inductance of any circuit, but in practice it is difficult unless the geometry is pretty simple. A solenoid is a device with a simple geometry and is widely used in electrical circuits. Let us, therefore, determine the self-inductance of a solenoid.

(b) Self-inductance of a Solenoid : *Consider a long solenoid of cross-sectional area A and length L, which consists of N turns of wire. To find its inductance we must relate the current in the solenoid to the magnetic flux through it. In previous lesson you have used Ampere's law to determine the magnetic field of a long solenoid, which is given as*

$$B = \mu_o n I \qquad ....(20.7a)$$

where n is the number of turns per unit length an I is the current through the solenoid. For our problem $n = N/L$, which gives

$$B = \frac{\mu_o N I}{L} \qquad ....(20.7b)$$

The total flux through the N turns of the solenoid is

$$\phi = N\,\mathbf{B} \int d\mathbf{S} = NB\,dS = NBA = \frac{\mu_o N^2 A}{L} \qquad ....(20.8)$$

Here, we have written **B**. d**S** = BdS, since the magnetic field of the solenoid is uniform and perpendicular to the cross-section of the individual turns. The self-inductance of the solenoid is

$$L = \frac{\phi}{I} \; \frac{\mu_o N^2 A}{L} \qquad ....(20.9)$$

Given this information you may like to determine the self-inductance and the back emf for a typical solenoid to get an idea of their magnitudes.

INTEXT QUESTIONS 20.3

1. *A solenoid 1m long and 20 cm in diameter contains 10,000 turns of wire. A current of 2.5 A flowing in it is reduced steadily to zero in 1.0 ms. What is the magnitude of the back emf of the inductor while the current is being switched off?*
   Take $\mu_o = 1.26 \times 10^{-26} Hm^{-1}$.

   ....

2. *A certain length (1) of wire folded into two parallel, adjacent strips of length L/2 is wound on to a cylindrical insulator to form a type of wire-wound non-inductive resistor (Fig. 20.9). Why can this configuration be called non-inductive?*

   ....

3. *What rate of change of current in a 9.7 mH solenoid produces a self-induced emf of 35mV?*

   ....

As we have said earlier, the back emf in an inductor opposes the change in current and its magnitude depends on how rapidly the current changes. If we try to stop current in a very short time, dI/dt is very large and a very large back emf appears. This is why switching off inductive devices, such as solenoids, can result in the destruction of delicate electronic devices by induced currents. Having worked out Intext Question 20.3 (1) you would realise that you have to be extremely cautious in closing switches in circuits containing large inductors. Even in your

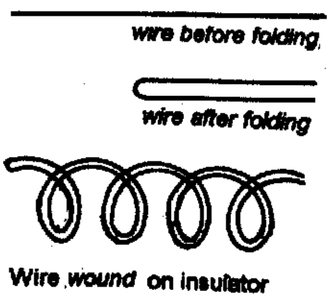


Fig. 20.9: Folded wire wound on to a cylindrical insulator. (a) wire before folding; (b) wire after folding, (c) wound wire.

day-to-day experience. You may have seen that you often draw a spark when you unplug an iron. Why does this happen? This is due to electromagnetic induction which tries to keep the current going, even if it has to jump the gap in the circuit. Now, you may wonder what happens when you plug in the iron and put on the switch? This bring us to the role of inductors in circuits. Let us consider the example of an *LR* circuit to understand this role.

20.4.2 LR Circuits

Suppose that a solenoid is connected to a battery through a switch (Fig. 20.10). Beginning at $t = 0$, when the switch is closed, the battery causes charge to move in the circuit. A solenoid has inductance (L) and resistance (R), and each of these influences the current in the circuit. The inductive and resistive effects of a solenoid are shown schematically in Fig.20.10. The symbol for inductance( ) is shown in series with the resistance symbol. For simplicity we assume that all of the resistance in the circuit, including the internal resistance of the battery, is represented by R. Similarly, L includes the self-inductance of the connecting wires. A circuit, such as that seen in Fig. 20.10, containing resistance and inductance in series is called an ***LR circuit***.

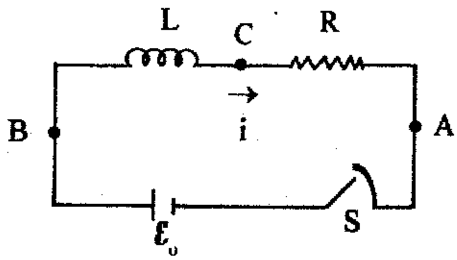


Fig. 20.10: An inductance L and resistance R are in series with a battery of emf o" Switch S is closed at t = o"

The role of the inductance in determining the current in the circuit can be understood qualitatively: As the current $i(t)$ in the circuit increases (from $i = 0$ and $t = 0$), there is a self-induced emf $\varepsilon = -L\, di/dt$ in the inductance whose sense is opposite to the sense of the increasing current. That is, since the current is increasing, the sense of the induced emf opposes that increase. This opposition to the increase in current prrevents the current from rising abruptly (prevents di/dt from becoming very large). On the other hand, if the current is changed negligibly (that is, di/dt is very small), there would be little opposition to a change in the current. The net effect of the inductance is to moderate between these extremes so that the current changes, but not too abruptly. The instantaneous value of the current depends on the values of L, R and the emf of the battery, ε_0. Let us now obtain these results quantitatively.

Consider once again the ciruit in Fig. 20.10 which has a battery (a source of constant emf ε_0) connected to a risistance R and an inductance L. What current flows in the circuit when we close the switch ?

Let us analyse this current quantitatively. The total emf in the circuit is the sum of the emf provided by the battery and the back emf of the inductor. Therefore, Ohm's law gives,

$$\varepsilon_0 - L\frac{dI}{dt} = IR \qquad ....(20.10)$$

On solving this equation we can obtain the expression for $I(t)$ as given below :

$$I(t) = \frac{\varepsilon_0}{R}(1 - e^{-Rt/L}) \qquad ....(20.11)$$

Had there been no inductance in the circuit, the current would have jumped immediately to ε_0/R. With an inductance in the circuit, the current rises gradually and reaches a steady state value of ε_0/R as $t \to 0$. The time it takes the current to reach about two-thirds of its steady state value is given by L/R, which is called the ***inductive time constant*** of the circuit. Significant changes in current in an LR circuit cannot occur on time scales much shorter that L/R. The plot of the current with time is shown in Fig. 20.11.

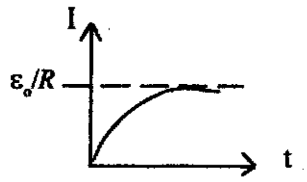


Fig. 20.11 : Current in an LR circuit

You can see that the greater the L is, the larger is the back emf, and the longer it takes the current to build up. Thus, the role of an inductance in electric circuits is somewhat similar to that of mass in machanical systems. You know that the larger the mass of an object is, the harder it is to change its velocity. In the same way, the greater L is in a circuit, the harder it is to change the current in the understanding of these ideas.

INTEXT QUESTIONS 6.4

(1) *A light bulb connected to a battery and a switch comes to full brightness nearly instantaneously when the switch is closed. However, if a large inductance is in series with the bulb, several seconds may pass before the bulb achieves full brightness. Explain why.*

. . . .

(2) *In an LR circuit how long does it take the current to reach half of its steady state value?*

. . . .

(3) *In an LR circuit, the current reaches 36 mA in 2.2 ms after the switch is closed. After sometime the current reaches it steady state value of 72 mA. If the resistance in the circuit is 68Ω. What is the value of the inductance?*

. . . .

Let us now consider the second situation in which the changing current in a circuit induces a current in an adjacent circuit. This is the phenomenon of mutual induction which gives rise to the proerty of mutual inductance.

20.4.3 Mutual Inductance

When the current changes in a coil in a circuit, there is a self-induced emf in the coil,

$$\varepsilon_L = -L\, di/dt.$$

As we have seen, the magnetic flux linking each turn of the coil is due to the magnetic field of the current in that circuit – hence the use of the term "self induced." The magnetic field also extends outside the coil and may influence another nearby circuit.

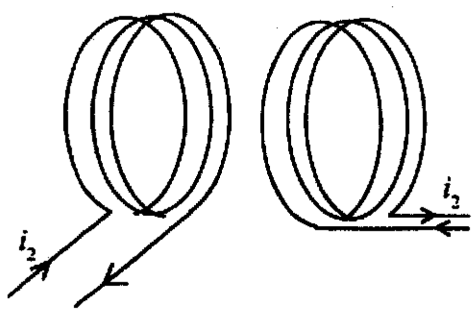


Fig. 20.12 : A changing current in each coil induces an emf in the other coil.

Fig. 20.12 shows stationary coils belonging to separate circuits. Each coil carries a current, and these currents and the magnetic field they produce can be changing. Thus the flux linking each turn of coil 2 changes because of the changing current in coil 1. Similarly, the flux linking each turn of coil 1 changes because of the changing current in coil 2. These contributions to the flux changes and the corresponding induced emfs are in addition to the self-induced contributions. Since induced emfs appear in each coil because of a change in the other coil, the interaction is mutual between the coils, and the effect is called ***mutual induction.***

Consider the induced emf in one of the coils say, coil 2, due to a change in the current i_1 in coil 1.The flux linking each turn of coil 2 has a contribution due to the magnetic-field contribution $\mathbf{B}_1$ of coil 1. We let the symbol ϕ_{21} represent the contribution of the magnetic field of coil 1 to the magnetic flux linking a turn in coil 2. It is this flux that changes when the current i_1 changes. We assume that the flux linking each of the N_2 turns is the same. The product $N_2\,\phi_{21}$ is called the ***number of flux linkages*** for the coil. If no magnetic material such as iron is around, the field contribution $\mathbf{B}_1$ is proportional to the current i_1 which produces it (from the Bio-Savart law). Then both ϕ_{21} and $N_2\phi_{21}$ are proportional to i_1. We write the linear relation between $N_2\,\phi_{21}$ and i_1 by introducing a constant M_{21}, a ***coefficient of mutual inductance:***

$$N_2\phi_{21} = M_{21}i_1 \qquad .....(20.12)$$

We now apply Faraday's law to determine the induced emf due to coil 1 in a turn of coil 2. The induced emf in each turn is $\varepsilon = -\mathrm{d}(\phi_{21})/\mathrm{d}t$, and the total emf ε_{21} induced in coil 2 is $N_2\varepsilon$:

$$\varepsilon_{21} = -N_2\frac{\mathrm{d}\phi_{21}}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(N_2\,\phi_{21}) = -\frac{\mathrm{d}}{\mathrm{d}t}(M_{21}i_1)$$

$$= -M_{21}\frac{\mathrm{d}i}{\mathrm{d}t} \qquad .....(20.13)$$

Thus, the induced emf ε_{21} in coil 2 is proportional to the rate of change of the current $\mathrm{d}i_1/\mathrm{d}t$ in coil 1. The coefficient M_{21} depends on geometrical factors such as the shapes of the coils, the way they are wound, and their relative separation and orientation. The minus sign is used to determine the sense of the induced emf from Lenz's law.

By reversing the roles of the two coils, we can consider the emf ε_{12} induced in coil 1 by the changing current i_2 in coil 2.The number of flux linkages for coil 1 is proportional to the current i_2, $N_1\phi_{12}=M_{12}i_2$, where M_{12} is a coefficient determined by the geometry. The induced emf in coil 1 is given by

$$\varepsilon_{12} = -M_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t} \qquad ....(20.14)$$

The coefficients M_{12} and M_{21} can be calculated easily for only a few simple arrangements. The values can be generally determined from measurements on the circuits. It turns out that the dependence on geometry of M_{12} and M_{21} is also mutual – that is, $M_{12} = M_{21}$. Thus, we can drop the subscripts and let M represent the ***mutual inductance*** of the pair. Then the emfs induced in each coil by the changing current in the other are given by

$$\boxed{\varepsilon_{12} = -M\frac{\mathrm{d}i_2}{\mathrm{d}t}\ , \ \varepsilon_{21} = -M\frac{\mathrm{d}i_1}{\mathrm{d}t}} \qquad ....(20.15)$$

This result applies generally to any pair or circuit elements. A changing current in each element induces an emf in the other element. The mutual inductance depends only on the geometry if no magnetic materials are nearby. The SI unit of mutual inductance is the henry (H), the same as the unit of self-inductance.

Example 20.3 : *A coil in one circuit is close to another coil in a separate circuit. The mutual inductance of the combination is 340 mH. During a 15-ms time interval, the current in coil 1 changes steadily from, 230 to 57 mA, and the current in coil 2 changes steadily from 36 to 16 mA. Determine the emf induced in each coil by the changing current in the other coil.*

Solution : During the 15-ms time interval, the currents in the coils change at the constant rates of

$$\frac{di_1}{dt} = \frac{57\text{ mA} - 23\text{ mA}}{15\text{ ms}} = 2.3\text{ A/s}$$

$$\frac{di_2}{dt} = \frac{16\text{ mA} - 36\text{ mA}}{15\text{ ms}} = -1.3\text{ A/s}$$

From Eq. 20.15, the magnitudes of the induced emfs are

$$\varepsilon_{21} = (340\text{ mH})\,(2.3\text{A/s}) = 0.77\text{ V}$$
$$\varepsilon_{12} = (340\text{ mH})\,(1.3\text{ A/s}) = 0.45\text{ V}$$

Remember that the minus signs in Eq. (6.15) refer to the sense of each induced emf.

You should now work out a few problems quickly.

20.5 SOME APPLICATIONS OF INDUCTANCE

One of the most important applications of the phenomenon of mutual inductance is the transformer about which you will study in lesson 21. Some other commonly used devices based on self-inductance are the choke coil and the ignition coil. We will discuss about these devices briefly.

Inductors can be used in combination with capacitors to produce currents that change direction with time at regular intervals. Such currents are called alternating currents. Let us consider LC circuits which give rise to periodically varying currents.

20.5.1 LC Circuits

Let us examine that behaviour of a circuit containing only a capacitance C and an inductance *L*, an ***LC circuit*** (Fig. 20.13). Ordinary circuits contain resistance, but for simplicity we begin by discussing an idealized circuit with negligible resistance. Suppose the capacitor in the circuit of Fig. 20.13 is charged by an external battery and then the battery is taken away. When the switch is closed, the capacitor will begin to discharge through the inductor so that at time there will be a current i in the circuit and a charge q on the capacitor. The charge and the current are related because the current gives the rate at which charge is transferred from one plate to the other: $i = \pm\, dq/dt$. The choice of plus or minus depends on our sign convention for i and q. Suppose we let i be positive when the current is clockwise and let q be positive when the charge on the upper plate is positive. With this choice, q increases when i is positive : $i = dq/dt$.

From Kirchhoff's loop rule, the sum of the potential differences around the loop is zero :

$$(V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_a - V_d) = 0$$

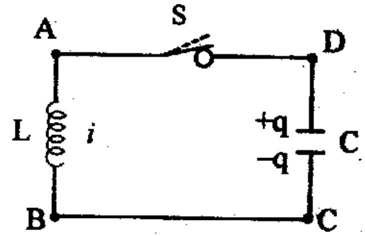


Fig.20.13 : An LC circuit

As $V_c = V_b$ and $V_d = V_a$ the potential difference across the inductor is $(V_b - V_a) = L(di/dt)$, where the algebraic sign is determined by Lenz's law. The induced emf opposes the change that causes it. Therefore, when $di/dt > 0$, $V_b > V_a$. With our sign convention for q, the potential difference across the capacitor is $V_d - V_c = q/C$. Thus, we have

$$L\frac{di}{dt} + \frac{q}{C} = 0 \qquad ....(20.16a)$$

Since $i = \frac{dq}{dt}$, we have $\frac{di}{dt} = \frac{d^2q}{dt^2}$ Making this substitution and re–arranging, we have

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q \qquad ....(20.16b)$$

This equation has the same mathematical form as the differential equation that describes a simple harmonic oscillator (see lesson 22 of book 4). An example of a simple harmonic oscillator is the one-dimensional spring-mass system where frictional forces are negligible. From our experience with the motion of a simple harmonic oscillator, we expect that the charge on the capacitor in the LC circuit varies sinusoidally with time :

$$q = q_m \cos(\omega_o t + \phi) \qquad ....(20.17a)$$

$$\text{with, } \omega_o = \frac{1}{\sqrt{LC}} \qquad ....(20.17b)$$

Here, q_m is he maximum charge on the capacitor, ω_o is the angular frequency of the oscillation, Φ is the phase constant, and $(\omega_o t + \phi)$ is the phase. Then the current is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}[q_m \cos(\omega_o t + \phi)] = -\omega_o q_m \sin(\omega_o t + \phi)$$

or

$$i = -i_m \sin(\omega_o t + \phi) \qquad ....(20.18)$$

Where $i_m = \omega_o q_m$ is the maximum current.

You can verify that Eq. (20.17) is a solutions of Eq. (20.16b). Notice that i varies with time with a period 2π. Thus, it is an alternating current.

Practically every day of our lives we use electrical devices that operate with alternating current. The list of such devices is long. Fans, lights, radios, TVs, telephones, refrigerators run on ac. So let us now learn more about alternating currents and voltages.

INTEXT QUESTIONS 20.5

1. *Consider the sense of the mutually induced emf's in Fig. 20.12 according to an observer located to the right of the coils. (a) At an instant when the current i_1 is increasing, is the sense of emf ε_{21} clockwise or conterclockwise ? (b) At an instant when i_2 is decreasing, is the sense of ε_{12} clockwise or counterclockwise?*

2. *Suppose that one of the coil in Fig.20.12 is rotated so that the axes of the coils are perpendicular rather than parallel. Would the mutual inductance remain the same, increase or decrease? Explain.*

20.6 ALTERNATING CURRENTS AND VOLTAGES

When a battery is connected to a resistor, charge flows through the resistor in only one direction. If we want to reverse the direction of the current, we must interchange the battery connections. If we could systematically interchange the battery connections, we would produce a current in the resistor that periodically changes direction. We call such a current an ***alternating current***.

Sources of potential difference whose polarity changes with time are called ***alternating sources***. The types of alternating potential differences are limited only by our imagination and ingenuity, and modern electronics exploits a number of different ones. For example, the saw-tooth potential difference shown in Fig. 20.14a is used across the horizontal deflector plates of a TV tube to sweep the electron beam back and forth across the screen. Among the many mathematical functions that describe alternating potential differences, $A \cos \omega t$ (or equivalently, $A \sin \omega t$), is particularly significant. Here, A is the amplitude, and ω is the angular frequency. Both A and ω are contant for a given physical situation. The angular frequency has the same interpretation as in simple harmonic motion. It is related to the frequency, ν, expressed in hertz, for example, by

$$\omega = 2\pi \nu$$

The function a sin ωt is shown in Fig.20.14 b. In general, alternating voltages and currents are mathematically expressed as

$$V = V_m \cos \omega t \quad \text{....(20.19a)}$$

$$I = I_m \cos \omega t \quad \text{....(20.19b)}$$

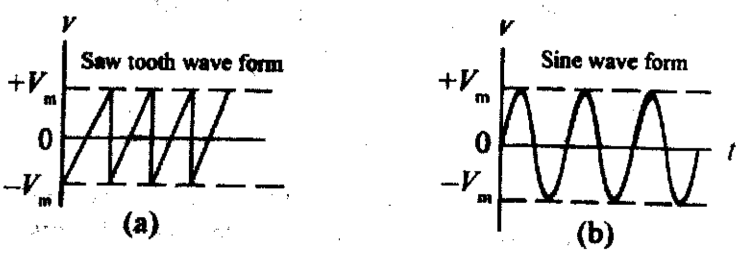


Fig. 20.14

Fig. 20.14 : (a) Saw tooth potential difference (b) Sinusoidal function which could represent either voltage or current

V_m and I_m are known as the ***peak values*** of the alternating voltage and current. In addition, we also define the root mean square values of V and I as

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad \text{....(20.20 a)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad \text{.....(20.20b)}$$

The relation between V_m and I_m depends on the circuit elements present in the circuit. Let us now study AC circuits containing (i) a resistor (ii) a capacitor, and (iii) an inductor only.

20.6.1 AC Source Connected to a Resistor

If we connect a resistor to an ac generator as shown in Fig. 20.15 we establish in the circuit a current that changes direction with time. The instantaneous value of the current is given by the instantaneous value of the potential difference across the resistor divided by the resistance.

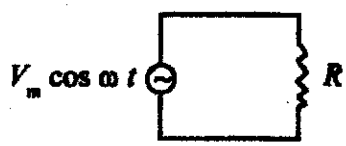


Fig.: 20.15 : An ac source connected to a resistor.

$$I = \frac{V}{R}$$

or $$I = \frac{V_m \cos \omega t}{R} \qquad \text{....(20.21a)}$$

The quantity V_m/R has units of volts per ohm, or amperes. It represents the maximum value of the current in the circuit. The current changes direction with time, and so we use positive and negative values of the current to represent the two possible current directions. Substituting I_m, the maximum current in the circuit, for V_m/R in Eq. (20.21a), we have

$$I = I_m \cos \omega t \qquad \text{....(20.21b)}$$

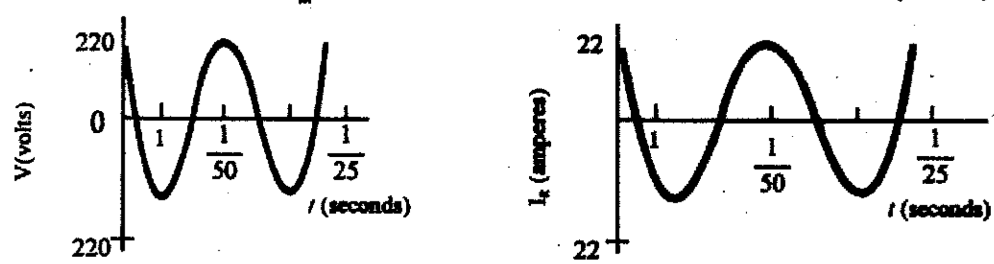


Fig. 20.16 : A graph of V and I with time in a circuit.

Fig 20.16 Shows the time variation of the potential difference between the ends of a resistor and the current in the resistor. Note that the potential difference and current are in phase– the peaks and valleys occur at the same time

In India, we have $V_m = 220V$ and $\nu = 50$ Hz, then for $R = 10$ ohms, we get

$$V = 220 \cos (2\pi\, 50t)\ V$$

and

$$I = \frac{220}{10}\cos(2\pi\, 50t)$$

$$= 22\cos(2\pi\, 50t)\ A$$

The cycles of V and I are shown in Fig. 20.16. Both V and I are proportional to cos (2π $50t$). Therefore, their peaks and valleys occur at the same time, and we say that they are in phase. The average current is zero over one, two, or any integral number of cycles.

The ***average thermal power*** developed in the resistor is not zero because the instantaneous power $P = I^2R$ is always positive (Fig. 20.17). Because I^2 is periodic, we can determine the average power, P_{av}, by considering a single cycle :

$$P_{av} = (I^2R)_{av} = R(I^2)_{av} = RI^2_m\,(\cos^2\omega t)_{av} \qquad ....(20.22)$$

For determining average values, we have

$$P_{av} = RI^2_m\left(\frac{\int_0^{2\pi}\cos^2\omega t\, d(\omega t)}{\int_0^{2\pi} d(\omega t)}\right)$$

or

$$P_{av} = R\left(\frac{I^2_m}{2}\right) \qquad .....(20..23)$$

Note that the same power would be produced by a constant dc current of $I_m/\sqrt{2}$ A in the resistor. It would also result if we were to connect the resistor to a potential difference having a constant value of $V_m/\sqrt{2}$ V. The quantities $I_m/\sqrt{2}$ and $V_m/\sqrt{2}$ are called the rms values of the current and potential difference. The term rms is short for root-mean-square, which means "the square root of the mean value of the square of the quantity of interest." For an electric outlet in an Indian home where $V_m = 220$V , the rms value of the potential difference is

$V_{rms} = V_m/\sqrt{2} \simeq 156$V .

This is the value generally quoted for the potential difference. Wouldn't you now like to do some exercises to understand these ideas better?

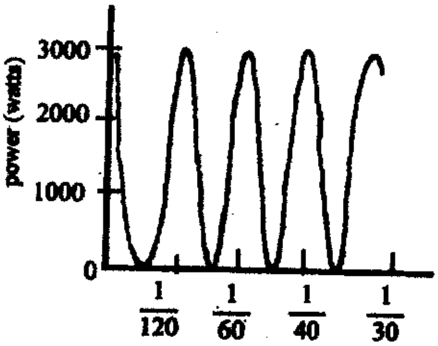


Fig. 20.17: The time variation of the thermal power developed in a resistor

INTEXT QUESTIONS 20.6

1. *In a light bulb connected to an ac source the instantaneous current is zero two times in each cycle of the current. Why is not the bulb off during these times of zero current?*

   ....

2. *An electric iron having a resistance 25Ω is connected to a 220 V, 50 Hz household outlet. Determine the average current, instantaneous current and the rms current in it.*

   ....

Let us now consider an ac circuit with a capacitor only.

20.6.2 AC Source Connected to a Capacitor

Fig. 20.18 shows a capacitor connected to an ac generator. We assume that the capacitor has been connected for a sufficiently long time to avoid any transient effect that might take place when the capacitor is first connected to the generator. The instantaneous charge on the capacitor equals the instantaneous potential difference across the capacitor multiplied by the capacitance ($q = CV$). This follows from the definition of capacitance.

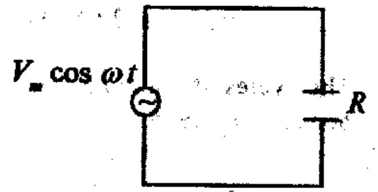


Fig. 20.18 : An ac generator connected to a capacitor

Thus,

$$q = C V_m \cos \omega t \quad ...(20.24)$$

Relating current and charge by $I = dq/dt$, we have

$$I = -\omega C V_m \sin \omega t \quad ...(20.25)$$

In order to compare V and I, let us again take the standard household values

$V_m = 220\text{V}$ and $\nu = 50\text{Hz}$.

Then, $\omega = 2\pi(50)$ rad /s
$= 314$ rad/s

and assume that $C = 10^{-5}$F. Then

$\omega = 314$ rad/s
$V = 220 \cos(2\pi\, 50t)$ V
$I = -220\,(2\pi\, 50)\, 10^{-5} \sin(2\pi 50t)$
$= -0.691 \sin(2\pi 50t)$ A

Two cycles of V and I are shown in Figure 20.19.

Unlike the situation for a resistor, the current I and potential difference V for a capacitor are ***not*** in phase. The first peak of the current plot occurs one quarter of a cycle before the first peak in the potential difference plot. Hence we say that the *capacitor current leads capacitor potential difference by one quarter of a period.* One quarter of a period corresponds to a phase difference of $\pi/2$, or 90°. Accordingly, we also say the potential difference lags the current by 90°.

Rewriting Eq.(20.25) as

$$I = -\frac{V_m}{1/(\omega C)} \sin \omega t \quad ....(20.26)$$

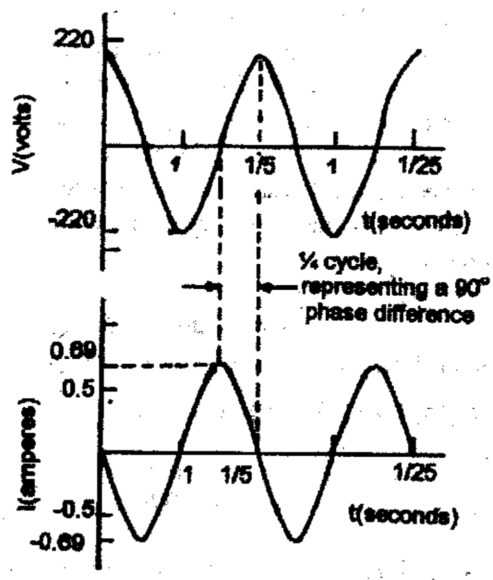


Fig. 20.19 : The time variation of the potential difference between the plates of a capacitor. Note that the potential difference and the current are not in phase–the peak in the current occurs one–quarter cycle before the corresponding potential difference peak.

and comparing Eqs.(20.21a) and (20.26),we see that ($1/\omega C$) must have units of resistance. The quantity $1/\omega C$ is called the ***capacitive reactance***, symbolized by X_c.

$$\boxed{X_c = \frac{1}{\omega C}} \qquad ....(20.27)$$

Capacitive reactance is a measure of how the capacitor limits the ac current in the circuit. It depends on both the size of the capacitance and the frequency of the generator. The capacitive reactance decreases if either frequency or capacitance increases. In the limit as the frequency goes to zero, the capacitive reactance becomes infinite. Zero frequency for the generator implies that there is a dc source, such as a battery. Because no charge actually flows between the plates of a capacitor, the infinite capacitive reactance for zero frequency in consistent with the behaviour of a capacitor connected to a dc source. Resistance and capacitive reactance are similar in the sense that both measure limitations to ac currents, but unlike resistance, capacitive reactance depends on the frequency of the ac generator. (Fig. 20.20) The concept of capacitive reactance allows us to introduce an equation analogous to the equation $I = V/R$ involving resistance R.

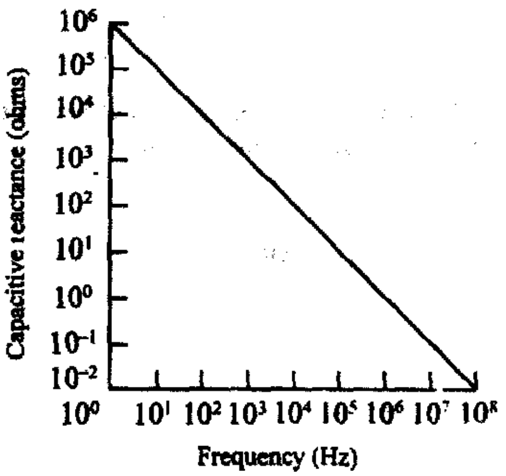


Fig. 6.20 : The reactance of a capacitor (C · 1 2r μF) as a function of the frequency measured in hertz. Note that the scales are logarithmic. The capacitive reactance decreases as the frequency increases.

$$I_{rms} = \frac{V_{rms}}{X_C} \qquad ...(20.28)$$

The instantaneous power delivered to the capacitor is the product of the instantaneous capacitor current and the potential difference.

$$P = VI$$
$$= -\omega C V^2 \sin \omega t \cos \omega t \qquad ..(20.29)$$
$$= -\frac{1}{2}\omega C V^2 \sin 2\omega t$$

The sign of P determines the direction of energy flow with time. When P is positive, energy is being stored in the electric field of the capacitor. When P is negative, energy is being released by the capacitor. Graphical representations of V, I, and P are shown in Fig. 20.21 . While both the current and the potential difference vary with angular frequency ω, the power varies with angular frequency 2ω. The average power is zero. The electric energy stored in the capacitor during a charging cycle is completely recovered when the capacitor discharges. On the average, then, there is no energy stored or lost in the capacitor in a cycle.

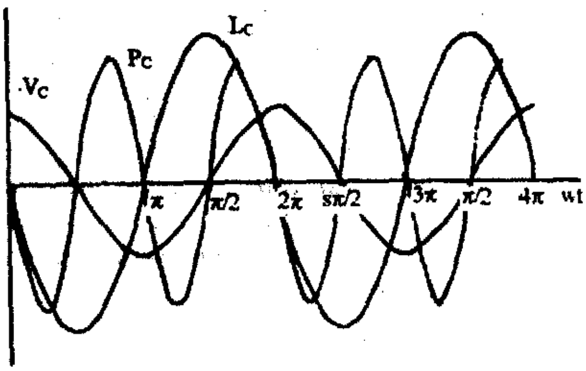


Fig. 6.21 : The time variation of the potential difference. current, and power for a capacitor connected to an ac generator

Example 6.5 : *A 100-μF capacitor is connected to a 50-Hz ac generator having a peak amplitude of 220 V. Let us determine the current recorded by an rms ac ammeter connected in series with the capacitor.*

Solution : At this frequency, the capacitive reactance of the capacitor is

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi\ (50)\ \frac{\text{rad}}{\text{s}}\ 100\times 10^{-6}\text{F}} = 31.8\ \Omega$$

Assuming that the ammeter has no effect on the current measurement, we have for the instantaneous current in the capacitor

$$I = \frac{-V}{X_C}\cos\ \omega t = -\frac{220}{31.8}\cos \omega t$$

$$= -\ 6.92\ \cos \omega t\ \text{A}$$

The rms current is,

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$= \frac{6.92}{\sqrt{2}}$$

$$= 4.91\ \text{A}$$

Now try the following exercises

INTEXT QUESTIONS 6.7

1. *Use your knowledge of how a capacitor functions to explain why the current in a capacitor-ac-generator circuit increases as the capacitance increases.*

   ....

2. *A capacitor is connected to an ac generator having a fixed peak value (V_m) but variable frequency. In terms of the manner in which a capacitor charges and discharges, why would you expect the current to iincreases as the frequency decreases?*

   ....

3. *Why would you expect the average power delivered to a capacitor by an ac generator to be zero?*

   ....

4. *Why do capacitative reactances become small in high-frequency circuits, such as those in a TV set?*

   ....

Finally let us discuss an ac circuit containing an inductor only.

20.6.3 AC Source Connected to an Inductor

We consider next an ideal (zero-resistance) inductor connected to an ac generator. (Fig. 20.22). Calling V the potential difference across the inductor, we can write :

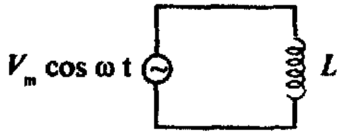


Fig. 20.22 : An inductor connected to an ac source.

$$V = L\frac{dI}{dt} = V_m \cos \omega t \qquad ....(20.30)$$

Integration of Eq. (20.30) results in

$$I = \frac{V_m}{\omega L} \sin \omega t + \text{constant} \qquad ...(20.31a)$$

The constant (of integration) represents a circuit current not depending on time. Because there is only an ac source, there is no time-independent current. Hence, the constant of integration is zero and we have

$$I = \frac{V_m}{\omega L} \sin \omega t \qquad ...(20.31b)$$

To compare V and I let us take $V_m = 220$V and $\omega = 2\pi(50)$rad/s, and assume that $L = 1H$.

Then,

$$V = 220 \cos (2\pi\, 50t) \text{ V}$$

$$I = \frac{220}{2\pi\,.\,50} \sin (2\pi 50t) = 0.701 \sin (2\pi 50t) \text{ A}$$

Two cycles of V and I are shown in Fig. 20.23 . The inductor current and potential difference are not in phase. Instead, the peaks and valleys of current and potential difference occur at different times. The first peak in the potential difference plot occurs one-quarter cycle before the first peak in the current plot. We say that the inductor current lags the inductor potential difference by $\pi/2$ rad (or 90°) . This is what we would expect from Lenz's law. Another way of seeing this is to rewrite Eq. (20.31b) as

$$I = \frac{V_m}{\omega L} \cos \left(\omega t - \frac{\pi}{2}\right)$$

Because $V = V_m \cos \omega t$, the $-\pi/2$ phase difference for I means that I lags V by $\pi/2$. This is in contrast to the current in a capacitor, which leads the potential difference. *For an inductor, the current lags the potential difference.*

The quantity ωL in Eq. (20.31b) has units of resistance and is called the ***inductive reactance***, symbolized by X_L.

$$X_L = \omega L \qquad ...(20.32)$$

As with capacitive reactance, X_C, inductive reactance, X_L, has units of ohms. ***Inductive reactance*** is a measure of how the inductor limits the ac current in the circuit. It depends on both the size of the inductance and the frequency of the generator. Inductive reactance increases if either frequency or inductance increases. This is just the opposite of capacitive reactance. In the limit as the frequency goes to zero, the inductive reactance goes to zero, in the same limit, capacitive reactance becomes infinite (see Table 20.11). Because inductive effects vanish for a dc source such as a battery, zero inductive reactance for zero frequency is consistent with the behaviour of an inductor connected to a dc source.

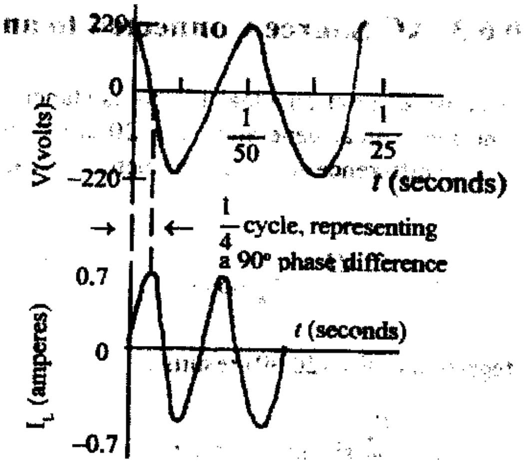


Fig.20.23 : The time variation of the potential difference betwcen the ends of an inductor and the current in the inductor. Note that the potential difference and current are not in phase–the peak in the current occurs one-quarter cycle after the corresponding potential difference peak.

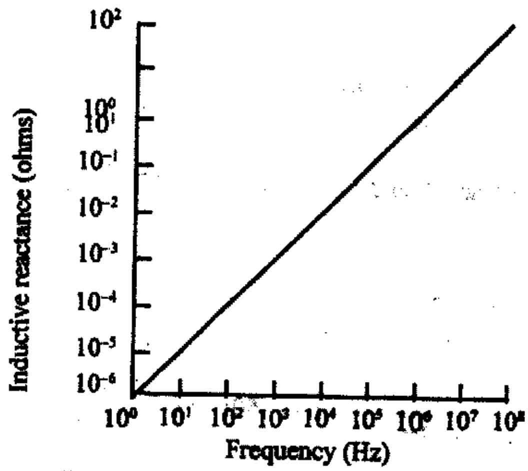


Fig. 20.24 : The reactance of an inductor ($L = 1/2\pi$ μH) as a function of the frequency measured in hertz. Note that the scales are logarithmic. The inductive reactance increases as the frequency decreases.

Table 20.1

	Reactance limit	*Low-Frequency limit*	*High-Frequency*
Resistor	$X_R = R$	R	R
Capacitor	$X_C = \frac{1}{\omega C}$	∞	0
Inductor	$X_L = \omega L$	0	∞

The concept of inductive reactance allows us to introduce an inductor analog to the equation $I = V/R$ involving resistance R.

$$I_{rms} = \frac{V_{rms}}{X_L} \qquad ...(20.33)$$

The instantaneous power delivered to the inductor is

$$P = VI$$

$$= \frac{V_m^2}{\omega L} \sin \omega t \cos \omega t = \frac{V_m^2}{2\,\omega L} \sin 2\,\omega t \qquad ....(20.34)$$

Graphical representations of V, I and P are shown in Fig. 20.25. Although both the current and the potential difference vary with angular frquency ω, the power varies with angular frequency 2ω. The average power delivered is zero. Energy is alternately stored and released as the magnetic field alternately grows and dwindles.

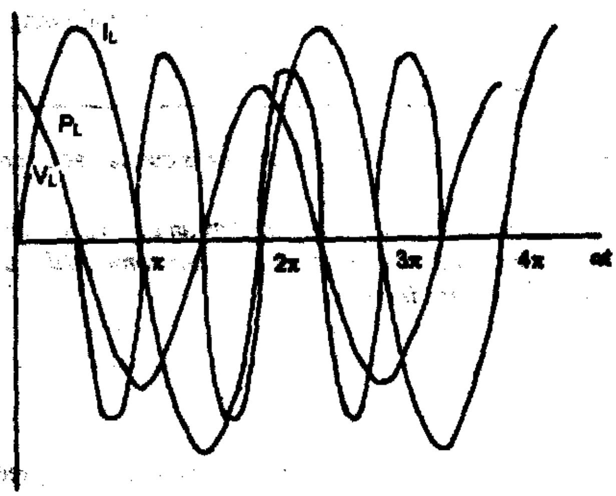


Fig. 20.25 : The time variation of the potential difference, current, and power for an ideal inductor connected to an ac generator.

Example 20.6 : *A certain solenoid with nothing in its interior has a length of 25cm and diameter of 2.5 cm, and contains 1000 closely spaced turns. The resistance of the coil was measured to be 1.00Ω. Compare the inductive reactance at 100 Hz with the resistance of the coil.*

lution : You know that the inductance of a solenoid whose length is large compared to diameter is,

$$L = \frac{\mu_o N^2 \pi A^2}{L}$$

where, N is the number of turns, A is the radius, and L is the length. Hence

$$L = \frac{(4\pi \times 10^{-7})\ \text{H/m}\ (1000)^2\ \pi\ (0.0125)^2\ \text{m}^2}{0.25\ \text{m}}$$

$$= 2.47 \times 10^{-3}\ \text{H}$$

The inductive reactance at a frequency of 100Hz is

$$X_L = \omega L = 2\pi\left(100\ \frac{\text{rad}}{\text{s}}\right)(2.47 \times 10^{-3})\text{H}$$

$$= 1.55\ \Omega$$

The inductive reactance of this solenoid at 100Hz is comparable to the intrinsic (ohmic) resistance. In a circuit diagram it would be shown as

You may now like to test your understand of these ideas. Try the following exercises.

INTEXT QUESTIONS 20.8

1. *Describe the role of Lenz's law when an ideal inductor is connected to an ac generator.*

   ....

2. *In section 20.3.1 self-inductance was characterized as an electrical inertia. Using this as a guide, why would you expect the current in an inductor connected to an ac generator to decrease as the self-inductance increases?*

   ....

20.6.4 LCR Circuits

Let us now conclude our discussion of impedance by considering all three elements – a resistor, an inductor, and a capacitor – connected to an ac generator (Fig. 20.26). Such circuits are very useful in electronics. Analysis of this circut is analogous to that for the RC and RL circuits. We write

$$V_m \cos \omega t = V_R + V_C + V_L$$

$$= R\frac{dq}{dt} + \frac{q}{C} + L\frac{d^2q}{dt^2} \qquad ...(20.35)$$

This equation is identical in form to that of a forced harmonic oscillator.

Assuming that,

$$q = q_m \sin(\omega t + \phi)$$

with q and ϕ undetermined parameters, we find that

$$q = \frac{V_m}{\omega Z}\sin(\omega t + \phi)$$

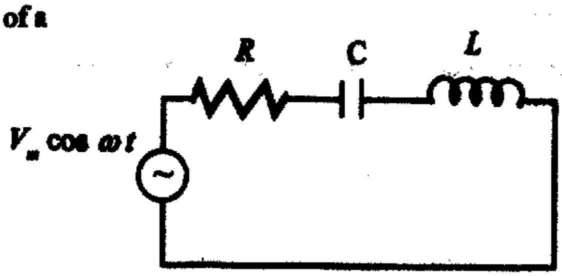


Fig.20.26 : An LCR circuit

Here, we define the impedance Z as

$$Z = \left[R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right]^{1/2}$$

$$= [R^2 + (X_C - X_L)^2]^{1/2} \qquad \text{....(20.36a)}$$

and

$$\tan\phi = \frac{X_C - X_L}{R} \qquad \text{...(20.36b)}$$

Fig. 20.27 shows the "useful" triangle for an RLC circuit. That X_L and X_C tend to cancel in the impedance equation is a consequence of the phase relations discussed earlier.

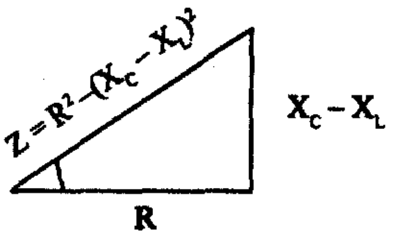


Fig. 20.27 : A "useful" triangle for an RLC circuit. The base is of "length" R and the altitude is of "length" $X_C - X_L$. The phase angle is then given by = tan $[(X_C - X_L)/R]$.

Note that if $X_L = 0$, meaning that there is no inductor in the circuit, then we arrive at the solution for an RC circuit. And if $X_C = 0$, meaning that there is no capacitor in the circuit, then we arrive at the solution we obtained for the RL circuit. The expressions for current and potential differences for this LCR circuit are identical in form to those for the RC and RL circuit.

$$I = \frac{V_m}{Z}\cos(\omega t + \phi) \qquad \text{...(20.37)}$$

$$V_R = \frac{V_m}{Z} R\cos(\omega t + \phi) \qquad \text{....(20.38)}$$

$$V_C = \frac{V_m}{Z} \times C\sin(\omega t + \phi) \qquad \text{....(20.39)}$$

$$V_L = \frac{V_m}{Z} \times L\sin(\omega t + \phi) \qquad \text{....(20.40)}$$

Consequently, the relations for the rms values for current and potential differences are unaltered:

$$I_{rms} = \frac{1}{\sqrt{2}}\,\frac{V_m}{Z} = \frac{I_m}{\sqrt{2}} \qquad \text{...(20.41)}$$

$$V_{rms} = \frac{1}{\sqrt{2}}\,\frac{V_m}{Z} R$$

$$= I_{rms} R \text{ for the resistor, and} \qquad \text{....(20.42)}$$

$$V_{rms} = \frac{1}{\sqrt{2}}\,\frac{V_m}{Z} X_C$$

$$= I_{rms} X_C \text{ for the capacitor,} \qquad \text{...(20.43)}$$

$$V_{rms} = \frac{1}{\sqrt{2}} \frac{V_m}{Z} . X_L$$

$$= I_{rms} X_L \text{ for the inductor.} \qquad ....(20.44)$$

These relations involving all three components also include the RC and RL circuits and the singly connected elements as special cases. For example, removing the inductor and capacitor is equivalent to setting $X_C = 0$ and $X_L = 0$. Then $Z = R$ and $\tan \phi = 0$, implying that $\phi = 0$, and we find that

$$V_R = V_m \cos \omega t$$

exactly as in Section 20.4.1 when only a resistor was connected to the ac generator.

Let us now calculate the power in an LCR circuit.

(a) Power in the LCR Circuit

You know that a capacitor connected to an ac generator reversibly stores and releases electric energy. There is no net energy delivered by the generator. Similarly, an inductor connected to an ac generator reversibly stores and releases magnetic energy. There is not net energy delivered by the generator. However, an ac generator delivers a net amount of energy when connected to a resistor. The energy is transformed into thermal energy in the resistor. When a resistor, an inductor, and a capacitor are connected in series with an ac generator, it is still only the resistor that causes a net energy transfer. We can confirm this by calculating the power delivered by the generator.

The instantaneous power is the product of the generator output and the current that results.

$$P = VI$$

$$= V_m \cos \omega t \left[\frac{V_m}{Z} \cos(\omega t + \phi) \right] \qquad ....(20.45)$$

The phase angle ϕ and angular frequency ω play important roles in the power delivered. If the impedancde Z is large at a particular angular frequency, then the power will be small for all values of the time. This is consistent with the idea that impedance measures how the combination of elements impedes (or limits) ac current.

$$\text{Average power} = \frac{V_m^2}{2Z} [\cos \phi + \sin (\omega t + q/2]$$

$$= \frac{V_m^2}{2Z} \cos \phi_2$$

(b) Resonance

An interesting and useful characteristic of the series RLC circuit driven by an ac source is the phenomenon of ***resonance***. Resonance is a common feature of systems that have a tendency to oscillate at a particular natural frequency. If such a system is driven by an energy source at a frequency that is near the natural frequency, then the amplitude of the oscillation is large. A familiar example is a child on a playground swing. The child seated on the swing has a natural frequency for swinging back and forth. If the child pulls on the ropes at regular intervals and the frequency of the pulls is almost the same as the natural frequency of swinging, then the amplitude of the swinging will be large.

Suppose we have a series LCR circuit driven by an ac source whose frequency can be varied. Consider the current amplitude I_m as we change the frequency while keeping other quantities (R, L, C and V_m) fixed.Eq. 20.37 shows that the current amplitude is limited by the impedance of the circuit, $I_m = V_m/Z$, where $Z = [R^2 + (X_L - X_C)^2]^{½}$ since $X_L = \omega L$ and $X_C = 1/\omega C$. There is a particular frequency at which $X_L = X_C$ so that the impedance is minimum at $Z = [R^2 + (0)^2]^{1/2} = R$. This angular frequency is represented by the symbol ω_o and is called the ***resonant angular frequency***.Using $X_L = X_C$ *at* $\omega = \omega_o$, we have $\omega_o L = 1/\omega_o C$ or

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}} \qquad ..(20.46)$$

When $\omega = \omega_0$,the current amplitude is maximum at $I_m = V_m/R$. Notice that ω_o is given by the same expression as the angular frequency of oscillation for the LC circuit with no resistor or source (Eq.20.17b) .It is often called the circuit's ***natural angular frequency.***

Fig. 20.28 shows graphs of I_m versus ω for two cases; in the upper curve, (i) $R = 100\Omega$, and in the lower curve, (ii) $R = 200\Omega$. The other quantities are the same for each curve : V_m = 100V, L = 1.00 mH, C = 1.00 nF. Each curve exhibits a maximum current at a resonant frequency. Since the product LC is the same for each case, the resonant frequency is also the same.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1.00\text{ mH})(1.00\text{ nF})} = 1.00 \text{ Mrad/s}.$$

At resonance , $I_m = V_m/R$. Thus the current amplitude at resonance is twice as great for the circuit with R =100 Ω compared with the circuit with R =200 Ω.At frequencies much less than ω_o, a circuit is predominantly capacitive, and the current is limited mainly by its capacitive reactance. At frequencies much greater than ω_o, a circuit is predominantly inductive, and the current is limited mainly by its inductive reactance. [here Mrad/s = Mega rad/s = 10^6 rad/s]

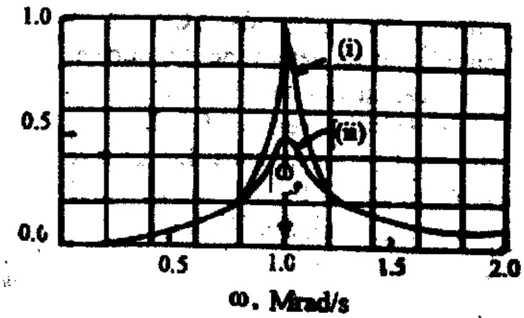


Fig. 20.28 : Current amplitude I_m versus ω for two cases ; (i) R = 100Ω and (ii) R = 200Ω Other quantities are V_m = 100 V, L = 1.00mH, and C = 1.00nF.

The tuning circuit of a radio or television set is an example of a circuit with a resonant frequency. The antenna of a radio accepts signals from many nearby stations. The antenna is the source in the tuning circuit, so the circuit is driven at many frequencies. However, the only large component of the current is the component that oscillates near the circuit's resonant frequency. The circuit discriminates against signals not near its resonant frequency. When you tune a radio, you vary the capacitance of a capacitor in the tuning circuit. This varies the resonant frequency of the circuit so that it matches the transmitting frequency of the station you wish to hear.

(e) Power Delivered at Resonance

You have just studied that the resonant frequency equals the natural frequency of oscillation of an LC circuit. Driving the circuit at the natural frequency of oscillation produces a maximum in the amplitude of the current. This situation is completely analogous to driving a machanical system at its natural frequency of oscillation in order to produce a maximum velocity. As observed in Eq. (20.45), the average power is always positive (or zero). The average power is zero when the generator frequency (ω) is zero, and approaches zero as the generator frequency becomes very larg ($\omega \rightarrow 0$). Thus between $\omega = 0$ and $0 \rightarrow$ a maximum occurs. The maximum power occurs for $\omega = \omega_e$, which is the same condition for maximum current. Evaluating Eq. (20.45) for the maximum power condition, we have

$$P_{max} = \frac{V^2_{rms}}{R} \qquad ...(20.47)$$

We now show briefly that it is advantageous to have $R^2\, C/L << 1$. Taking $R^2\, C/L = 10^{-3}$, we obtain the average power delivered by the generator for different values of the angular frequency ω shown in Fig. 20.29.

The power falls off sharply at frequencies above or below the resonant frequency ω_e. The mountain-like appearance of the power versus frequency plot inspires its name, the *resonance* peak. A narrow resonance peak means that power is efficiently delivered to the circuit for only a very narrow range of frequencies.

(d) Tuning a Radio

As we said earlier, a radio is tuned in to a particular station by the adjustment of a capacitive reactance or an inductive reactance to achieve a resonance condition in a resonant circuit. For a particular LCR tuning circuit, let us determine the current produced at resonance when an rms potential difference of 0.05 V is provided by the antenna. Let us take $R = 150\ \Omega$, $L = 15$ mH, and $C = 3.45$ pF. The resonant frequency ω_e for this *LCR* circuit is

$$\omega_e = \frac{1}{\sqrt{LC}}$$

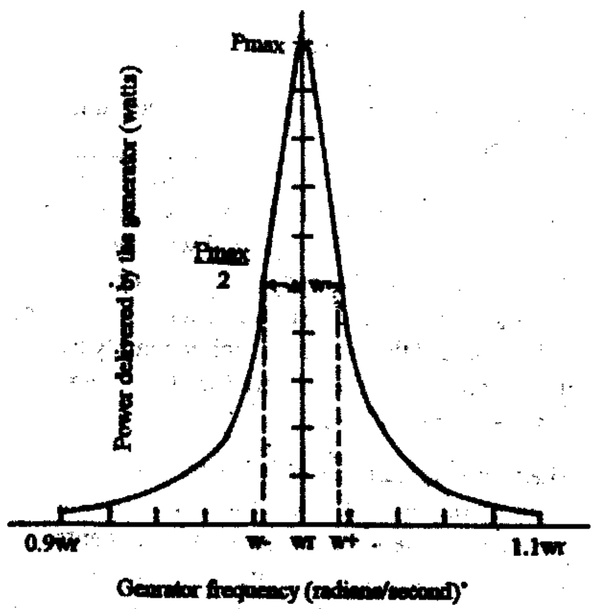


Fig. 20.29. : Power delivered to an LCR circuit as a function of the angular frequency ω of the generator for the situation $R^2\, C/L = 10^{-3}$. The power delivered is a maximum when the generator frequency equals the resonant frequency of the circuit, $\omega = \omega_e = 1/\sqrt{LC}$. For a generator frequency 10% less or 10% greater than the resonant frequency the power delivered is about 97% less than the maximum.

$$= \frac{1}{(15 \times 10^{-3}\,H \times 3.45 \times 10^{-12}\,F)^{1/2}}$$

$$= 4.40 \times 10^{6}\ \text{rad /s}$$

then

$$v_0 = \frac{\omega_0}{2\pi}$$

$$= 7.00 \times 10^{5}\text{Hz}$$

This frequency lies in the amplitude modulated broadcast band. At resonance the impedance equals the circuit resistance $Z = R = 150\ \Omega$. Hence the rms current is,

$$I_{rms} = \frac{V_{rms}}{R}$$

$$= \frac{0.05\ V}{150} = 333\ \mu A$$

To see how the current changes for a slightly different frequency, let us compute the current in the same circuit for an rms input of 0.05 V at a frequency of 7.2×10^{5} Hz. First we determine the impedance at this frequency.

$$X_L = \omega L$$

$$= 2\pi \times 7.2 \times 10^{5}\ \text{rad/s} \times 15 \times 10^{-3}\text{H}$$

$$= 6.786 \times 10^{4}\ \Omega$$

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{2\pi \times 7.2 \times 10^{5}\,\text{rad/s} \times 3.45 \times 10^{-12}\text{F}}$$

$$= 6.407 \times 10^{4}\ \Omega$$

$$Z = [R^2 + (X_C - X_L)^2]^{1/2}$$

$$= [\,150^2 + (6.407 \times 10^{4} - 6.785 \times 10^{4})^2]^{1/2}$$

$$= 3793\ \Omega$$

Then, the rms current for this non-resonant frequency is

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{0.05\text{V}}{3793\text{W}}$$

$$= 13.2\ \mu A$$

A 0.05 V signal at the resonant frequency of 7.0×10^{5}HZ produces a current of 333 μA. A 0.05 V signal at a frequency of 7.2×10^{5}HZ produces only 13.2 μA when connected to the same LCR circuit. There is a sharp response of the circuit at the resonant frequency.

20.7 WHAT YOU HAVE LEARNT

- An emf or a current is induced in a conducting surface if the magnetic flux linking the surface changes, this is the phenomenon of ***electromagnetic induction.***
- The induced emf ε in a single loop is given by ***Faraday's law:***

$$\varepsilon = -\frac{d\phi_B}{dt}$$

where ϕ_B is the magnetic flux linking the loop.
- According to ***Lenz's Law,*** the sense of the induced emf (induced current) is such as to oppose the change in flux which produces it.
- If the current changes in a circuit element, such as a coil, a self-induced emf exists.
- For a long, tightly wound solenoid of length ℓ_o cross – section A having N number of turns it is given by

$$L = \frac{\mu_o N^2 A}{L_o}$$

- The current in an LR circuit with a source of constant emf ε_o is given by :

$$I(t) = \frac{\varepsilon_o}{R}(1 - e^{-Rt/L})$$

Where L/R is the inductive time constant of the circuit.
- The changing currents in two near by coils mutually induce an emf in each other.
- In an LC circuit, the charge on the capacitor and the current in the circuit oscillate sinusoidally with the same angular frequency ω_o.

$$\omega_o = \frac{1}{\sqrt{LC}}$$

- In a circuit with an a.c. source, the voltage across the source is given by $V = V_m \cos \omega t$ and $I = I_m (\omega t + \phi)$
- In a purely resistive circuit $\phi = 0$, the ac voltage and current are in phase,

The average power in such a circuit is $P_{av} = \frac{I_m^2 R}{2}$

- In a purely ***capacitive circuit,*** $\phi = +\frac{\pi}{2}$. Thus, the current leads the voltage by a phase of 90°. The average power in such a circuit is zero .
- In a purely inductive circuit, $\phi = -\frac{\pi}{2}$ and the current lags the voltage by a phase of 90°. The average power in such a circuit is zero.
- In a series LCR circuit connected to an ac source $I_m = \frac{V_m}{Z} = \frac{V_m}{[R^2 + (X_L - X_C)^2]^{1/2}}$ where Z is the impedance of the circuit : $Z = [R^2 + (X_L - X_C)^2]^{1/2}$
- If the angular frequency ω of the source is the same as the circuit's resonant angular frequency $\omega_o = 1/\sqrt{LC}$, then $X_L - X_C = 0$ and the impedance has a minimum value $Z = R$ with a maximum current $I_m = V_m/R$.

The average power is $P_{av} = V_{rms} \cdot I_{rms} = (I^2_{rms}) R$

20.8.6 TERMINAL QUESTIONS

1. Each loop in a 250-turn coil has face area $S = 9.0 \times 10^{-2}\ m^2$. (a) What is the rate of change of the flux linking each turn of the coil if the induced emf in the coil is 7.5 V? (b) If the flux is due to a uniform magnetic field at 45° from the axis of the coil, what must be the rate of change of the field to induce that emf?

2. a) In Fig. 20.30a what is the direction of the induced current in the loop when the area of the loop is decreased by pulling on it with the forces labelled F? B is directed into the page and perpendicular to it.

   b) What is the direction of the induced current in the smaller loop of Fig. 20.30 b when a clockwise current as seen from the left is suddenly established in the larger loop, by a battery not shown?

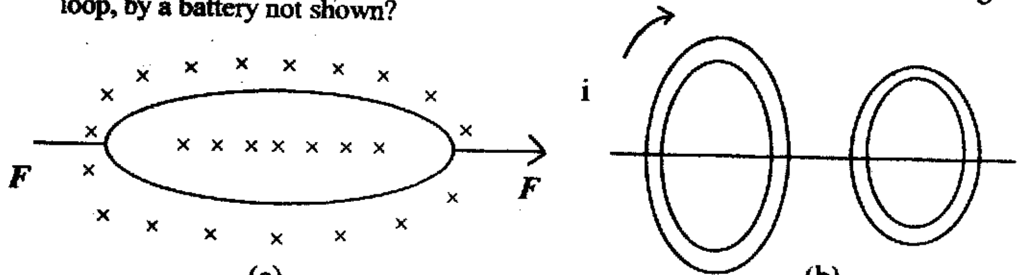


Fig. 20.30

3. a) If the number of turns in a solenoid is doubled, by what amount will its self-inductance change?

   b) Petrol in a vehicle's engine is ignited when a high voltage applied to a spark plug causes a spark to jump between two conductors of the plug. This high voltage is provided by an ignition coil, which is an arrangement of two coils wound tightly one on top of the other. Current from the vehicle's battery flows through the coil with fewer turns. This current is interrupted periodically by a switch. The sudden change in current induces a large emf in the coil with more turns, and this emf drives the spark. A typical ignition coil draws a current of 3.0 A and supplies an emf of 24 kV to the spark plugs. If the current in the coil is interrupted every 0.10 ms, what is the mutual inductance of the ignition coil?

4. a) Why is the rms value of an ac current always less than its peak value?
   b) The current in a 2.5 μF capacitor connector to an ac source is given by

   $I = -4.71 \sin 377\, t\ \mu A$

   Determine the maximum voltage across the capacitor.

5.a) Calculate the capacitive reactance (for $C = 2\ \mu F$) and the inductive reactance (for $L = 2mH$) at (i) 25 Hz and (ii) 50Hz.

b) Calculate the maximum and rms currents in a 22 μH inductor connected to a 5V (rms) 100 MHz generator.

6. A series RLC circuit with $R = 580\Omega$, $L = 31$ mH, and $C = 47$ nF is driven by an ac source. The amplitude and angular frequency of the source are 65 V and 33 krad/s. Determine (a) the reactance of the capacitor, (b) the reactance of the inductor, (c) the impedance of the circuit, (d) the phase difference between the voltage across the source and the current, and (e) the current amplitude, (f) does the current lead or lag the voltage across the source?

7. What is electromagnetic induction ? Explain the Faraday's laws of electromagnetic induction.

8. State Lenz's law. Show that Lenz's law is a consequence of law of conservation of energy.

9. What is self - induction ? Explain the physical significance of self - inductance.

10. Distinguish between the self-inductance and mutual-inductance. On what factors do they depend?

11. What is the source of induced e.m.f. in electromagnetic induction? Explain.

12. How much e.m.f. will be induced in a 10 H inductor in which the current charges from 10A to 7 A in 9×10^{-2} s.?

13. Explain why the reactance of a capacitor decreases with increasing frequency, whereas the reactance of an inductor increases with increasing frequency ?

14. What is impedance of an LCR series circuit ? Derive an expression for power dissipated in a.c. LCR circuit.