23

ELASTIC WAVES

23.1 INTRODUCTION

In our daily life we observe that material particles and objects are in motion. When a stone is dropped into the calm water in a pond, we observe concentric rings of alternate elevations and depressions emerging out from the point of impact and spreading on the surface of water. Consider a straw piece floating on the surface of water. You will observe that the straw piece moves up and down. It also moves back and forth. Here is something other than a particle which moves. We call it a ***wave.***

In this age we are well aware of the fact that radio waves carry sound from one place to another with the speed of light. These waves like light waves do not need a material medium for propagation. However, we shall restrict our discussion in this lesson about the mechanical waves propagating in elastic media.

23.2 OBJECTIVE

After studying this lesson, you should be able to,

- *explain propagation of transverse and longitudinal waves and establish the relation* $v = \nu \lambda$*;*
- *derive the relation for the velocity of a longitudinal waves in a gas and to explain, Laplace's correction, and to discuss the factors on which this velocity depends;*
- *derive an expression for the velocity of a transverse sound wave on a stretched string;*
- *derive the equation of a simple harmonic wave;*
- *explain the principle of superposition of the waves, explain the phenomena of beats, interference, phase change of a wave on reflection from a denser medium;*
- *explain formation of stationary waves or standing waves and explain the terms node and antinodes;*

- *discuss harmonics of organ pipes and stretched strings;*
- *discuss Doppler effect and its applications;*
- *explain characteristics of musical sound, intensity of sound waves, its unit and threshold intensity; and*
- *discuss noise pollution and shock waves.*

23.3 WAVE PROPAGATION

Let us take a long coil spring called slinky or a long thick rubber tube. Tie its one end to the stem of a tree or handle of a door. Hold its free end in your hand, give it a jerk downward. (Fig. 23.1). You will observe that a 'kink' is produced which travels towards the fixed end with definite speed. This kink is a wave of short duration. Go on moving the free end continuously up and down you will observe a wave train travelling towards the fixed end.

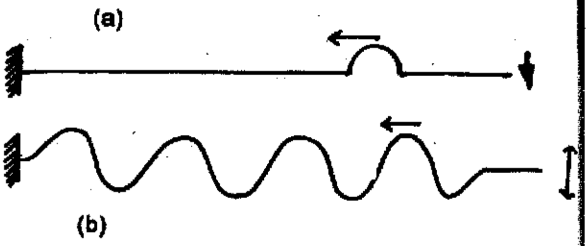


Fig. 23.1:(a) Wave pulse on a rubber tube or a slinky. (b) a continuous wave train on a rubber tube or a slinky

23.3.1 Propagation of Transverse Waves

Let us consider (Fig.23.2) a row of spherical balls of equal masses, evenly spaced and connected together by string pieces. Let us imagine that by means of a suitable device, ball 1, from left, is made to execute S.H.M. in a direction perpendicular to the row of balls with a period T. All the balls, owing to inertia of rest will not begin to oscillate at the same time. The motion will be handed over from one ball to the next after a certain time. Let us suppose that the time taken by the disturbance to travel from one ball to the next is $T/4$s. Initially, that is, at t = 0, all the balls are at rest and occupy the positions shown in Fig. 23.2. After $t = T/4$s ball 1 will have maximum displacement in the positive direction and the disturbance will just arrive at ball 2 (Fig. 23.2 ii). After $t = T/2$s, ball 1 will be at its mean position, ball 2 will have maximum displacement in the positive direction and ball 3 will just start moving up (Fig. 23.2 iii). After $t = 3T/4$ s, ball 3 has maximum displacement in the negative direction, ball 2 is in its mean position, ball 3 has maximum displacement in the positive direction and the disturbance just arrives

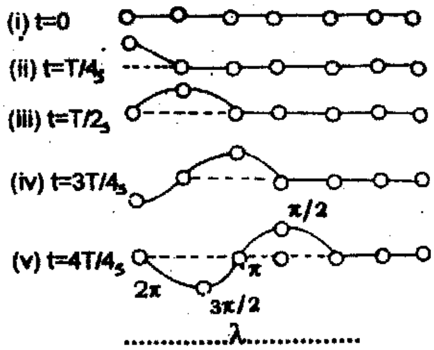


Fig. 23.2: *Propagation of a transverse wave*

at ball 4. After $t = 4T/4$ s the positions of the balls is as shown in Fig. 23.2 (iv). In this manner the disturbance moves ahead. Let us note the phase angles of the five balls at $t = T$s. The phase angle of ball no.1 is 2π that of ball 2 is $3\pi/2$, that of ball 3 is π and ball No. 4, 5, have phase angles of $\pi/2$ and zero respectively. Note carefully that ball 1 and 5 at this time are in the mean position and at the point of moving upwards. We say that ball 1 and 5 are vibrating in the same phase.

In a wave motion the distance between the two nearest particles vibrating in the same phase is called a wavelength denoted by λ.

It is evident that time taken by the wave to travel a distance $\lambda = T$. (See fig. 23.2). Therefore the velocity of the wave is

$$\frac{\text{Distance}}{\text{Time}} = \nu = \frac{\lambda}{T} \tag{23.1}$$

But $1/T = \nu$, the cyclic frequency. Therefore,

$$\upsilon = \nu\lambda \tag{23.2}$$

Further, as the phase difference between the two vibrating particles in the same phase is λ, therefore, the phase change per unit distance,

$$k = \frac{2\pi}{\lambda} \tag{23.3}$$

We call '*k*' the propagation constant. You remember that ω is the phase change per unit time. But the phase change in time *T*, is 2π hence

$$\omega = \frac{2\pi}{T} = 2\pi\nu \tag{23.4}$$

Dividing eq. (23.3) by eq. (23.4) we get

$$\frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda}$$

or

$$\frac{\omega}{k} = \nu\lambda = \upsilon$$

the velocity of the wave. Thus,

$$\upsilon = \frac{\omega}{k} = \nu\lambda \tag{23.5}$$

Let us now explain how the logitudinal waves propagate. Again we have to think of mechanical model.

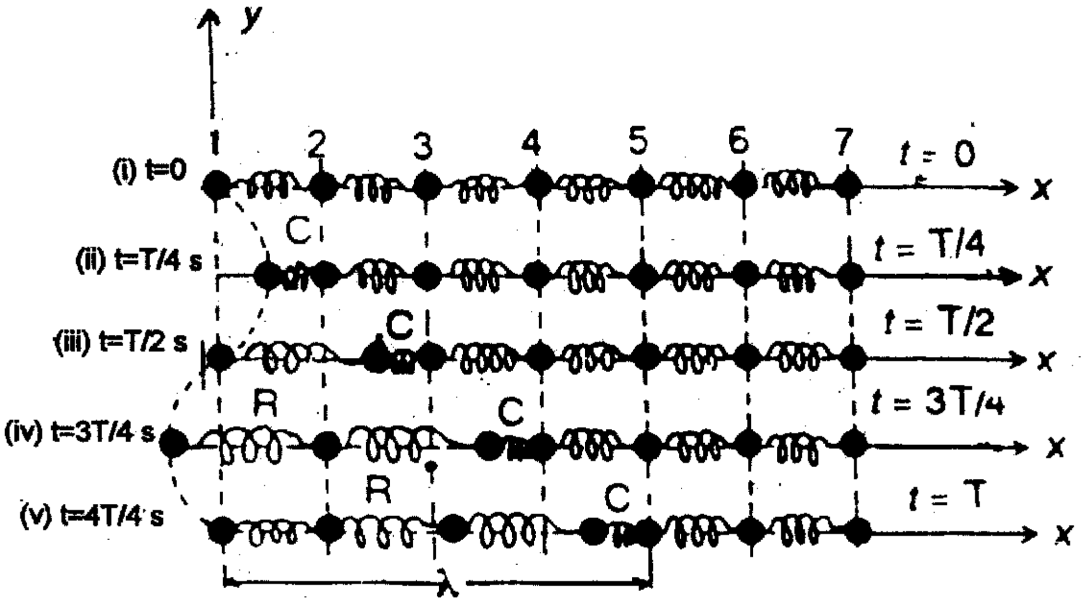


Fig. 23.3: *Propagation of longitudinal waves*

23.3.2 Propagation of Longitudinal Waves

Let us consider a row of balls (Fig. 23.3 (i) of equal masses, equally spaced and connected by means of spring pieces of equal force constant. By a suitable device ball No.1 in the row from left is made to execute simple harmonic vibration parallel to the row of balls. Let us suppose that T is the time period of vibration of each ball. After a time $t = T/4$ ball No.1 will have maximum displacement in the positive direction and the disturbance will just arrive at ball 2 [Fig. 23.3 (ii)]. Now the spring piece between ball 1 and 2 expands. As a result, ball 1 moves toward left and ball 2 moves towards right. After $t = T/2$, ball 1 returns to its original position and 2 has a maximum displacement towards right [Fig. 23.3 (iii)] and the others are unaffected so far. As ball 1 returns to its initial position; owing to inertia of motion, it overshoots and has a maximum displacement on the negative direction [Fig. 23.3(iv)]. After $4T/4$, the positions of the balls is as shown in Fig. 23.3(v). In this position ball 1 and 5 are in the opposite phases of vibration but ball 1 and 3 are in the same phase of vibration. The distance between 1 and 3 is λ, the wavelength.

We define, ***wavelength as the distance between the two nearest particles vibrating in the same phase***. Note that ball 1 and 3 are passing through their mean positions and moving in the opposite directions. Hence, the phase difference between them is λ .

As the phase difference between ball 1 and 3 is 2π and the distance between them is λ, hence the phase change per unit distance is $k = 2\pi/\lambda$, where k is known as *propagation constant*. We get all other relations same as obtained in the previous sub-section for transverse waves.

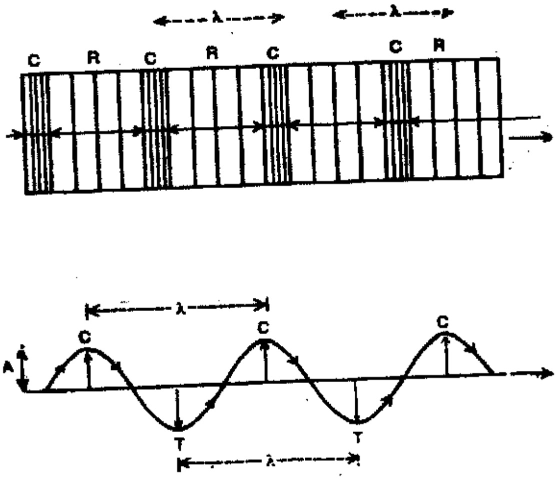


Fig. 23.4: *(i) Longitudinal wave depicting* λ*, (ii) transverse wave depicting* λ.

The wavelength is shown in Fig. 23.4 for both longitudinal and transverse waves. Let us now derive the equation of a simple harmonic wave.

23.3.3 Equation of a Simple Harmonic Wave in One Dimension

When we speak, sound waves are produced in three dimensions, the light waves emanating from a point source too travel in all directions, the waves produced on the surface of water are waves in two dimension. But the waves produced on a coil spring or on a string are waves in one dimension. We shall restrict our discussion to the waves in one dimension only.

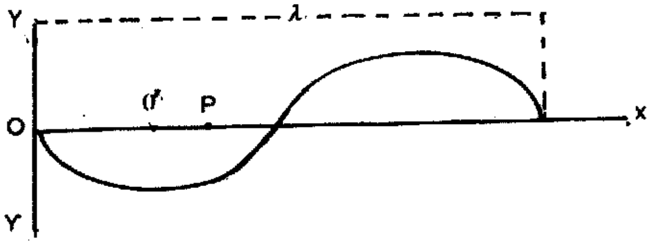


Fig. 23.5: *Simple harmonic wave travelling along x-direction*

Let us consider a simple harmonic wave propagating along OX (Fig. (23.5). We shall assume that the wave is transverse and the vibrations of the particle are along YOY. Let

$$y = a \sin \omega t. \tag{23.6}$$

represent the vibrations at a given instant, $t = 0$ at the point O. Then the vibrations at that time at the point P lags behind by a phase angle say ϕ. Thus, the vibrations at that time at the point P is given by

$$y = a \sin (\omega t - \phi) \tag{23.7}$$

Let us put OP = x. Since phase change per unit distance is k, therefore $\phi = kx$ Hence,

$$y = a \sin (\omega t - kx) \tag{23.8}$$

Further as $\omega = 2\pi/T$ and $k = 2\pi/\lambda$, therefore,

$$y = a \sin \left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)$$

or $$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \tag{23.9}$$

or $$y = a \sin \frac{2\pi}{\lambda} \left(\frac{\lambda}{T} t - x\right)$$

or $$y = a \sin \frac{2\pi}{\lambda} (vt - x) \tag{23.10}$$

Phase difference between two waves :

In deriving eq. (23.8) we have assumed initial phase of the wave, at O is zero. However, if the initial phase angle at O is ϕ_o then the equation of the wave would be

$$y = a \sin [(\omega t - kx) + \phi_o] \tag{23.11}$$

Thus, if two simple harmonic waves travelling along OX are represented by the equations

$$y = a \sin (\omega t - kx) \tag{23.8}$$

and $$y = a \sin [\omega t - k (x + \Delta x)] \tag{23.12}$$

then the phase difference between them is

$$\phi = k\Delta x = \frac{2\pi}{\lambda} . \Delta x \tag{23.13}$$

where Δx is called the path difference

Remember that phase angle of a wave is the angle whose sine or cosine times the amplitude gives the displacement y at a given location at a given time and the phase difference between the two waves at a given point equals to $2\pi/\lambda$ times the path difference at that point.

23.3.4 Transverse Versus Longitudinal Waves

Now we shall be summarising the points of difference between transverse and longitudinal waves.

Transverse waves	**Longitudinal waves**
(i) Displacements of the particles are perpendicular to the direction of propagation of the wave.	(i) Displacements of the particles are parallel to the direction of propagation of the wave.
(ii) Transverse waves look as crests and troughs propagating in the medium.	(ii) Longitudinal waves give the appearance of alternate compressions and rarefaction moving forward.
(iii) Transverse waves can only be transmitted in solids or on the surface of the liquids.	(iii) Longitudinal waves can travel in solids, liquids and gases.
(iv) In case of a transverse waves, the displacement - distance graph is the actual picture of the wave itself.	(iv) In case of longitudinal waves, the graph only represents the displacement of the particles at different points at a given time.

(a) Essential properties of the medium for propagation of longitudinal and transverse mechanical waves are given below:

These are (i) the particles of the medium must possess mass, (ii) the medium must possess elasticity. Longitudinal waves for propagation in a medium require volume elasticity but transverse waves need shape modulus or modulus of rigidity. However, light waves and other electromagnetic waves which are essentially transverse do not need any material medium for their propagation.

(b) Energy transmission by travelling waves : Let ρ be the density of the medium, *a*, the amplitude of the particles, and ω be the angular frequency of the particles of the medium. The maximum velocity of an oscillating particle is $v_0 = a\omega$. Therefore, maximum kinetic energy of a particle of the medium is $1/2\ m\, v_0^2 = 1/2\ m a^2\omega^2$, where *m* is the mass of the particle. If υ be the wave velocity then the volume of the medium that the wave sweeps in 1 s when it moves normally across a unit area is *V*. The intensity *I* of the wave is the energy that flows per second across a unit area, therefore,

$$I = \tfrac{1}{2}\rho v a^2\omega^2 = 2\pi^2\rho v a^2 V^2 \qquad (23.14)$$

Thus, the intensity of the wave is proportional to the square of the frequency of the wave and the square of the amplitude of the wave.

INTEXT QUESTIONS 23.1

1. *State the difference between longitudinal and transverse waves?*

....

2. *State the relation between phase difference and path difference.*

....

3. *Two simple harmonic waves are represented by the equations* $y = a \sin (\omega t - kx)$ *and* $y = a \sin [(\omega t - kx) + \phi]$, *what is the phase difference between the two waves?*

....

4. *What is meant by the intensity of the wave. State the factors on which the intensity of a wave depends.*

....

23.4 VELOCITY OF LONGITUDINAL WAVES IN AN ELASTIC MEDIUM

Newton's formula for the velocity of a longitudinal wave in an elastic medium of infinite extent is $v = \sqrt{\frac{E}{\rho}}$ (23.15)

Where, E = elasticity of the medium.

23.4.1 Newton's Formula for Velocity of Sound in a Gas

Newton assumed that compression and rarefaction caused by the sound waves during their passage through a gas take place under isothermal condition. This means that the changes in volume and pressure take place at constant temperature. Under such conditions the velocity of sound wave in a gas was given by Newton as,

$$v = \sqrt{\frac{P}{\rho}} \qquad (23.16)$$

For air, at S.T.P, that is at standard pressure and temperature,

$$P = 1.01 \times 10^5 \text{ Nm}^{-2} \text{ and } \rho = 1.29 \text{ kg m}^{-3}$$

On substituting these values in eq. (23.16) we get

$$v = \sqrt{1.01 \times 10^5/1.29} = 280 \text{ ms}^{-1}$$

When a gun is fired at the top of a distant mountain, we first see the smoke and a few seconds thereafter hear the sound. The reason is that the velocity of light is very much greater than the velocity of sound in air. By measuring the time interval between observing the smoke and hearing the sound, the velocity of sound in air can be determined. Using an improved technique, the velocity of sound in air has been determined as 333 m s^{-1} at 0°C. The difference in the value predicted by Newton's formula and that determined experimentally is $(333-280)/333 \times 100 = 16\%$. This error is too high to be regarded as an experimental error. Obviously there is something wrong with Newton's assumption that during the passage of sound, the compression and the rarefaction of air take place isothermally.

23.4.2 Laplace's Correction

Laplace pointed out that the changes in pressure of air layers caused by passage of sound take place under adiabatic condition owing to the following reasons.

(i) Air is bad conductor of heat;

(ii) Compression and rarefactions caused by the sound are too rapid to permit heat to flow out during compression and allow heat to flow in during rarefaction.

Under adiabatic conditions

$$E = \gamma P$$

Hence,

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad (23.17)$$

For air $\gamma = 1.4$, therefore, at STP

$$v = \sqrt{1.4 \times 1.01 \times 10^5/1.29} = 333 \text{ ms}^{-1}$$

INTEXT QUESTIONS 23.2

1. *What was the assumption made by Newton in deriving his formula?*

....

2. *What was wrong with Newton's formula?*

....

3. *Show that for every 1°C rise in temperature, the velocity of sound in air increases by 0.61* ms^{-1}.

....

4. *Find the temperature at which the velocity in air is* $\frac{3}{2}$ *times the velocity of sound at 7°C?*

....

23.5 VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING

We shall now derive an expression for the velocity of a transverse wave along a stretched (taut) string. Consider a wave pulse (Fig. 23.6 (a) travelling along a stretched string with a velocity v in a direction from left to right. The string along with the wave pulse is moved with a velocity v in a direction from right to left so that the wave pulse remains stationary relative to an observer on the Earth. We shall focus our attention on the part pq of the wave pulse. The part pq can be regarded as an arc of a circle. A magnified form of the arc pq shown in Fig. 23.6 (b). The point O is the centre of the circular are pq and θ is the angle subtend by the half arc at O. The forces acting on the arc pq are T, T. The tension of the string in the direction of arrows. Each of the two forces T, T is resolved into two components $T\sin\theta$ towards PO and $T\cos\theta$ perpendicular to PO.

The point P is the middle point of the arc. The forces $T\cos\theta$ and $T\cos\theta$ perpendicular to OP cancel out; but the forces $T\sin\theta$ and $T\sin\theta$ having a resultant equal to $2T\sin\theta$ provide the necessary centripetal force for the element of the wire in the arc pq to move in a circular path of radius r = OP. If m be the mass per unit length of the wire and δl be the length of the arc pq, then

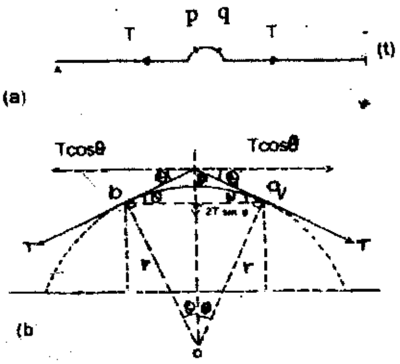


Fig. 23.6(a): *Transverse wave pulse travelling from left to right on a string (b) Forces acting on the element pq*

$$2T\sin\theta = m\delta l\, v^2/r.$$

As θ is small, therefore $\sin\theta \cong \theta$. Further $\delta l = 2\theta$, therefore,

$$2\,T\theta = mv^2\,2\theta$$

or $$\boxed{v = \sqrt{T/m}} \qquad (23.18)$$

INTEXT QUESTIONS 23.3

1. *What is the formula for the velocity of a wave on stretched string?*

   ....

2. *If λ be the wavelength of the wave and n be the frequency of the wave then what would be the relation between n, λ, T and m?* Further if $\lambda = 2l$, what would be the relation between n, l, T and m ?

   ....

23.6 SUPERPOSITION OF WAVES

Suppose two wave pulses travel in the opposite directions on a rubber tube or a sliky. What happens when they meet and thereafter? Do they knock each other out? To answer these questions, let us consider an experiment.

Produce two wave crests on a stretched thick rubber tube, or slinky, as shown in Fig. 23.7 and watch carefully what happens. The two crests are moving in the opposite directions. They meet and overlap at the point midway between them (Fig. 23.7(b) and then separate out. Thereafter, each one moves in the same direction in which it was moving before crossing and each one is having the same shape as it was having before crossing (Fig. 23.7 (c)).

Now produce one crest and one trough on the rubber tube or slinky as shown in Fig. 23.7 (d). The two are moving in the opposite directions. They meet (Fig. 23.7 (e), overlap and then separate out. Each one moves in the same direction in which it was moving before crossing and each one has the same shape as it was having before crossing. Now let us find out what happens when the two pulses overlap [(Fig. 23.7 (e) and (f)].

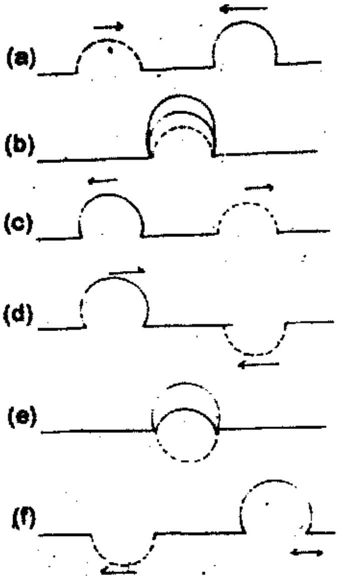


Fig. 23.7: *Illustrating principle of superposition of waves.*

At the points where the two pulses overlap, the resultant displacement is the vector sum of the displacements due to each of the two wave pulses. This is called the ***principle of superposition.***

The above experiment not only demonstrates the principle of superposition but also shows that two or more waves can travers the same space independent of each other. Each one travels as if the other waves were not present. This important property of the waves enable us to tune to a particular radio station as if the programme at other frequencies of the other stations was not being broadcast. Hereinafter we shall make use of this principle to explain the phenomena of interference of waves, formation of beats and stationary or standing waves.

INTEXT QUESTIONS 23.4

1. *When two waves travelling in the opposite direction meet, what happens then and thereafter?*

   ....

2. *What happens when two marbles each of the same mass travelling with the same velocity along the same line meet?*

   ....

3. *Two similar wave pulses travelling in the opposite directions on a string meet. What happens (i) when the waves are in the same phase? (ii) the waves are in the opposite phases?*

   ....

23.6.1 Reflection and Transmission of Waves

We shall confine our discussion in respect of mechanical waves produced on strings and springs. What happens and why does it happen when a transverse wave crest propagates towards the fixed end of a string? Only the experiment will provide the answer. Let us, therefore, perform the following activity.

Activity 23.1 : Fasten one end of a thick rubber tube to a fixed support (Fig. 23.8 (a) keeping the rubber tube horizontal, give a jerk to its free end so as to produce a transverse wave pulse travelling towards the fixed end of the tube (Fig. 23.8 (a)). You will observe that the pulse bounces back from the fixed end. As it bounces back, the crest becomes a trough which travels back in the opposite direction. Why? As the pulse meets the fixed end, it exerts a force on the support. The equal and opposite reaction not only reverses the direction of propagation of the wave pulse but also reverses the direction of the displacement of the wave pulse (Fig. 23.8 (b)). The support being much heavier than the tube, can be regarded as a denser medium. The wave pulse moving in the opposite direction is called the reflected wave pulse. So ***when reflection takes place from a denser medium, the wave undergoes a phase change of π , that is, it suffers a phase reversal.***

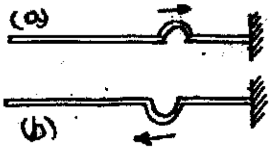


Fig. 23.8 (a): *Reflection from a denser medium (b) wave suffers a phase reversal*

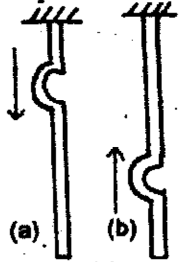


Fig. 23.9 (a): *A pulse travelling down towards the free end, (b) On reflection from free end direction of its displacement remains unchanged*

Let us now see what happens on reflection from a rarer medium. Only experiment can provide the reason and as such let us perform another activity.

Activity 2 : Suspend a fixed rubber tube from a fixed support (Fig. 23.9 a). Then send a wave-pulse travelling down the tube. On reflection from the free end, the wave pulse travels upward but without any change in the direction of its displacements. Why? As the wave pulse reaches the free end of the tube, it causes the free end to overshoot. Hence there is no change in the direction of displacement of the wave pulse. Note that air is rarer than the rubber tube. Thus on ***reflection from a rarer medium, no phase change takes place.***

Fig. 23.10: *Longitudinal waves are reflected from a denser medium without change of type but with change of sign*

Now the question arises : Do the logitudinal waves too behave similarly? Watch a row of bogies arranged as shown in Fig. 23.10(a). Now suppose that the engine E moves a bit towards the right. The buffer spring between the engine E and the first bogie in the row gets compressed. As this compressed spring expands, the spring between the 1st and the 2nd bogie gets compressed. As the second compressed spring expands, it moves a bit towards the 3rd bogie. In this manner the compression arrives at the last buffer spring in contact with the fixed stand D. As the spring between the fixed stand and the last bogie expands, only the last bogie moves towards the left. As a result of this, the buffer spring between the next two bogies on left is compressed. This process continues, till the compression reaches between the engine and the first bogie on its right. Thus, a compression returns as a compression. But the bogies then move towards the left. The buffer spring and the bogies form a medium. The bogies are the particles of the medium and the spring between them shows the forces of elasticity.

Thus, when reflection takes place from a denser medium, the longitudinal waves are relfected without change of type but with change in sign. And on reflection from a rare medium a logitudinal wave is reflected back without change of sign but with change of type. By 'change of type' we mean that rarefaction is reflected back as compression and a compression is reflected back as rarefaction.

INTEXT QUESTIONS 23.5

1. *What happens when a transverse wave pulse travelling along a string meets the fixed end of the string?*

   ....

2. *What happens when a wave pulse travelling along a string meets the free end of the string?*

   ....

3. *What happens when a wave of compression is reflected from (i) a rarer medium (ii) a denser medium?*

   ....

23.6.2 Interference of waves

Let us compute the ratio of maximum and minimum intensities in an interference pattern obtained due to superposition of waves. Consider two simple harmonic waves of amplitudes a_1 and a_2 each of angular frequency ω, both propagating along ox, with the same velocity $v = \omega/k$ but differing in phase by a constant phase angle ϕ. If

$$y_1 = a_1 \sin(\omega t - kx) \qquad (23.19)$$

be the equation of one wave, then the equation of the other wave is

$$y_2 = a_2 \sin[(\omega t - kx) + \phi] \qquad (23.20)$$

where $\omega = 2\pi/T$ and $k = \frac{2\pi}{\lambda}$ and ϕ is the phase difference between the two waves.

Since, the two waves are travelling in the same direction with the same velocity along the same line they overlap. According to the principle of superposition the resultant displacement at the given location at the given times

$$y = y_1 + y_2 = a_1 \sin(\omega t - kx)$$

$$+ a_2 \sin[(\omega t - kx) + \phi]$$

If we put $(\omega t - kx) = \theta$, then

$$y = a_1 \sin\theta + a_2 \sin(\theta + \phi)$$

or $$y = a_1 \sin\theta + a_2 \sin\theta \cos\phi +$$

$$a_2 \cos\theta \sin\phi$$

or $$y = (a_1 + a_2 \cos\phi) \sin\theta +$$

$$a_2 \sin\phi \cos\theta$$

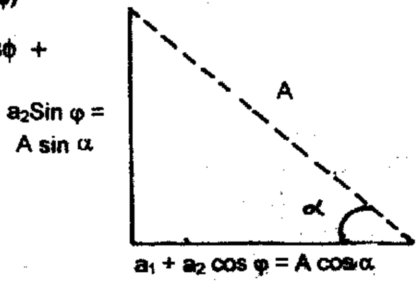


Fig. 23.11: *Calculating resultant amplitude A.*

Let us put $a_2 \sin\phi = A \sin\alpha$ and

$$a_1 + a_2 \cos\phi = A \cos\alpha$$

Then $y = A \cos\alpha \sin\theta + A \sin\alpha \cos\theta$

$$= A \sin(\theta + \alpha)$$

Substituting for θ we get

or $$y = A \sin[(\omega t - kx) + \alpha]$$

Thus, the resultant wave is of angular frequency ω and has an amplitude A given by

$$A^2 = (a_1 + a_2 \cos\phi)^2 + (a_2 \sin\theta)^2 = a_1^2 + a_2^2 \cos^2\phi + 2a_1a_2 \cos\phi$$

$$2a_1 a_2 \qquad \cos\phi + a_2^2 \sin^2\phi$$

or $$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi \tag{23.21}$$

In eq.(23.21) ϕ is the phase difference between the two supperposed waves. If p is the path difference between the two waves, then the phase difference between them is

$\frac{2\pi p}{\lambda}$, where $\frac{2\pi}{\lambda}$ is the phase change per unit distance.

When the path difference $p = 2m\,\lambda/2$, that is an even multiple of $\frac{\lambda}{2}$ then $\phi = 2\pi/\lambda \times 2m\,\lambda/2 = 2m\pi$. Since $\cos 2\pi = +1$, therefore, from eq. (23.2) we get

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2 \tag{23.22}$$

Hence, the amplitude and the intensity of the resultant wave is maximum.

From eq. (23.22) we have

$$I = I_{max} = (a_1 + a_2)^2$$

When $p = (2m + 1)\lambda/2$, then $\phi = (2m + 1)\pi$ and $\cos\phi = -1$. Then from eq. (23.21)

$$A^2 = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)$$

Hence, then $I = I_{min} = (a_1 - a_2)^2$.

Thus $\frac{I_{max}}{I_{max}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$

23.6.3 Beats

We have seen that superposition of waves of the same frequency propagating in the same direction produces interference. Let us now investigate what would be the outcome of superposition of waves of nearly the same frequency. First let us perform an activity.

Activity : Take two tuning forks of same frequency 512 Hz. Let us name them as A and B. Load the prong of the tuning fork B with a little wax. Now sound them together by a rubber hammer. Press their stems against a table top and note what you observe. You will observe that the intensity of sound alternately becomes maximum and minimum. These alternations of maxima and minima of intensity are called beats. One alternation of a maximum and a minimum is one beat. On loading the prong of B with a little more wax, you will find that beats become more rapid. On further loading the prongs of B no beats will be heard. The reason is that our ear is unable to hear two sounds as separate produced in an interval less than one tenths of a second. Let us now explain how beats are produced.

(a) Production of beats : Let us suppose we have two tuning forks A and B of frequencies N and $N + n$ respectively; n is smaller than 10. In one second, A completes N vibrations but B completes $N + n$ vibrations. Thus, in 1s B completes n more vibrations than the tuning fork A does. In other words, B gains n vibrations over A in 1s and hence it gains1 vib. in (1/n) s. and half vibration over A in (1/2n) s. Suppose at $t = 0$ s, i.e. initially, both the tuning forks were vibrating in the same phase. Then after (1/2n)s, B will gain half vibration over A. Thus, after 1s the tuning forks A and B will vibrate in the opposite phase. If A sends a wave of compression then B sends a wave of rarefaction to the observer. Hence, the resultant intensity as received by the ear would be zero. After (1/n) s, B would gain one complete vibration over A and hence the two forks will be again in the same phase of vibration. If now A sends a wave of compression. B too would send a wave of compression to the observer. The intensity observed would become maximum. After (3/2n)s, the two forks again vibrate in the opposite phase and hence the intensity would again become minimum. This process would continue. The observer would hear 1 beat in (1/n) s, and hence n beats in 1s. Thus, the *number of beats heard in 1s equals to the difference in the frequencies of the two tuning forks.* If more that 10 beats are produced in 1s, the beats would not be heard as separate. The beat frequency is n and beat period is $1/n$.

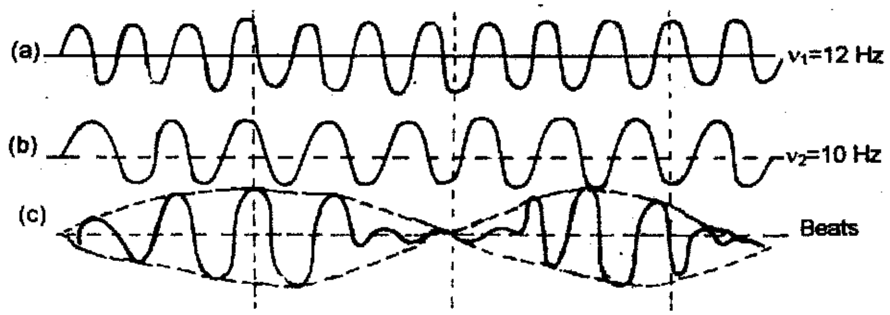


Fig. 23.12: *(a) Displacement time graph of frequency 12 Hz. (b) displcement time graph of frequency 10 Hz. Superposition of the two waves produces 2 beats per second.*

(b) Graphic method : Draw a 12 cm long line. Divide it into 12 equal parts of 1 cm. On this line draw 12 wavelengths each 1 cm long and height 0.5 cm. This represents a wave of frequency 12 Hz. On the line (b) draw 10 wavelengths each of length 1.2 cm and height 0.5 cm. This represents a wave of frequency 10 Hz. (c) represents the resultant wave. Fig. 23.12 is not the actual waves but the displacement time graphs. Thus, the resultant intensity alternately becomes maximum and minimum. The number of beats produced in 1 second is $\Delta\gamma$. Hence the beat frequency equal to the difference between the frequencies of the waves superposed.

INTEXT QUESTIONS 23.6

1. *If the intensity ratio of two waves is 1:16, and they produce interference, what will be the ratio I_{max}/I_{min}?*

   ....

2. *What happens when waves emanating from two sources of sound are superposed when their frequencies are ν and $\nu + 4$?*

   ....

3. *Two waves of frequencies ν and $\nu + \Delta\nu$ are supperposed, what would be the frequency of beats?*

   ....

4. *Two tuning forks A and B produce 5 beats per second. On loading one prong of A with a small ring, again 5 beats per second are produced. What was the frequency of A before loading if that of B is 512 Hz. Give reason for your answer.*

   ....

23.7 SUPERPOSITION OF WAVES OF SAME FREQUENCY TRAVELLING IN THE OPPOSITE DIRECTIONS

So far we have discussed travelling waves. In such waves, under particular conditions of the medium, crest and troughs are formed or rarefactions and of the compressions medium travel forward with a velocity depending upon certain properties of the medium. You will now study stationary or standing waves. In these waves crest and troughs or compressions and rarefaction remain stationary relative to the observer.

23.7.1 Formation of Stationary or Standing Waves

Let us explain the formation of stationary/standing waves. By now it should be clear to you that these waves are formed due to superposition of two waves of the same wavelength, same amplitude travelling with the same speed along the same line in the same medium but in the opposite directions. To understand their formation refer to Fig. 23.13 where we have shown the positions of the incident, reflected and resultant waves, each after $T/4$ s that is, after quarter of a period of vibration.

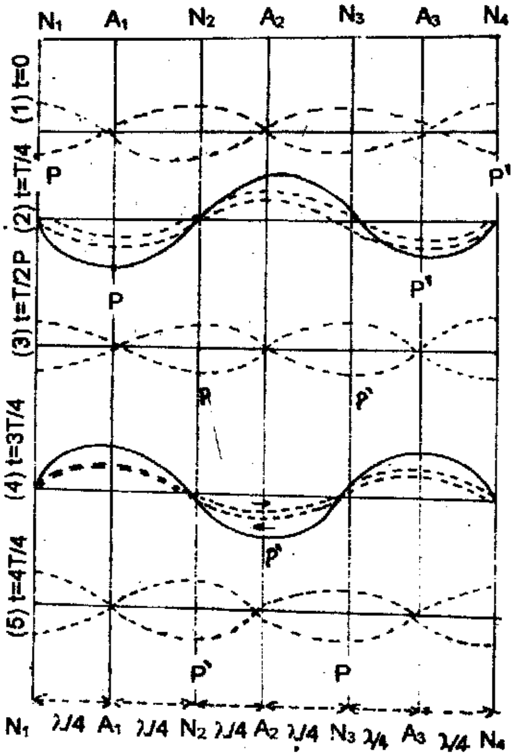


Fig. 23.13: *Showing formation of stationary waves due to superposition of two waves of same wave length, same amplitude travelling in opposite direction along the same line*

(i) Initially, at $t = 0$, [Fig. 23.13 (i)] the incident wave, shown by dotted curve, and the reflected wave, shown by dashed curve, are in the opposite phases. Hence the resultant displacement at each point is zero. All the particles are in their respective mean positions.

(ii) At $t = T/4$s [Fig. 23.13 (ii)], the incident wave has advanced to the right by $\lambda/4$, as shown by the shift of the point P and the reflected wave has advanced to the left by $\lambda/4$ as shown by the shift of the point P'. The resultant wave form has been shown by the thick continuous curve. It can be seen that the resultant displacement at each point is maximum. Hence the particle velocity at each point is zero and the strain is maximum.

(iii) At $t = T/2$s [Fig. 23.13(iii)]. The incident wave has advanced a distance $\lambda/2$ to the right as shown by the shift of the point P and the reflected wave has advanced a distance $\lambda/2$ to the left as shown by the shift of the point P'. At each point the displacements being in the opposite directions, have a zero resultant shown by a thick line.

(iv) At $t = 3T/4$ s [Fig. 23.13(iv)]. The two waves are again in the same phase. The resultant displacement at each point is maximum. The particle velocity is zero but the strain is maximum possible.

(v) At $t = 4T/4$ s [Fig. 23.13(v)] the incident and reflected waves at each point are in the opposite phases. The resultant wave form is a straight line (shown by an unbroken thick line). The strain $\Delta y/\Delta x$ at each point is zero.

It is to be noted :

(i) That at points N_1, N_2, N_3 and N_4, the amplitude is zero but the strain is maximum. Such points are called **nodes**.

(ii) that at points A_1, A_2 and A_3, the amplitude is maximum but the strain is minimum. These points are called **antinodes**.

(iii) That the distance between two successive nodes or between two, successive antinode is $\lambda/2$.

(iv) That the distance between a node and next antinode is $\lambda/4$.

(v) the time period of oscillation of a stationary wave equals the time period of the two travelling waves whose superposition has resulted in formation of the stationary wave.

(vi) the energy alternately surges back and forth about a point but on the average the energy flow past a point is zero.

The two similar waves travelling with the same speed in the opposite directions on the same line overlap. Due to their superposition stationary waves are produced. They are called stationary waves, because the wave form does not move forward, but alternately shrinks and dialates.The energy merely surges back and forth and on the average the flow of energy past a point is zero.

23.7.2 Equation of Stationary Waves

The equation of a simple harmonic wave travelling with a velocity $v = \omega/k$ in a medium

$$y_1 = -a \sin(\omega t - kx)$$

On reflection from a denser medium let the wave travel along the same line that is ox in the opposite direction with phase change of π. The equation of the reflected wave is therefore,

$$y_2 = a \sin(\omega t + kx)$$

Thus, owing to the superposition of the two waves the resultant displacement at a given point and time is

$$y = y_1 + y_2$$

Therefore, $y = a \sin(\omega t - kx) - a \sin(\omega t + kx)$

Let us use the trigonometric identity.

$$\sin A - \sin B = 2 \sin (A - B)/2. \cos (A + B)/2$$

which gives $y = -2a \sin kx \cos \omega t$ (23.23)

Let us put $-2a \sin kx = A$, then

$$y = A \cos \omega t \qquad (23.24)$$

Eq. (23.23) represents the resultant wave of angular frequency ω but of amplitude $A = -2a \sin kx$ at a location $x = x$. This is the equation of stationary simple harmonic wave. The amplitude of the resultant wave, which is A, oscillates in space with an angular frequency $k = 2\pi/\lambda$ which is the phase change per metre. At such points where

$$kx = m\,\pi \text{ or } x = m\,\pi/k = m\pi/\frac{2\pi}{\lambda} = m\,\lambda/2 \qquad (23.25)$$

$$\sin kx = \sin m\,\pi = 0. \text{ Hence } A = 0$$

These points where the amplitude is zero are known as **nodes**. At these points $\Delta y/\Delta x$ = maximum, that is strain is maximum. Obviously the spacing between two nearest points is $\lambda/2$.

At those points where $kx = (2\,m + 1)\,\pi/2$

or $x = (2m + 1)\,\lambda/2\ x\ \lambda/2\pi = (2m + 1)\,\lambda/4$

$\sin kx = \sin (2m + 1)\,\pi/2 = \pm 1$. Hence A is maximum. At these points the strain $\Delta y/\Delta x$ is zero. Obviously the spacing between two such neighbouring points is $\lambda/2$. These points where the amplitude is maximum but strain is zero are called ***antinodes***.

It may be pointed out, that at nodes where the strain is maximum the particle velocity is zero and at antinodes where the strain is zero the particle velocity $\Delta y/\ \Delta t$ = max. Therefore, it follows that the average flow of energy across any point is zero. The energy merely surges back and forth. Hence, these waves have been named stationary or standing waves.

23.7.3 Distinction between Travelling and Standing Waves

Let us summarise the main points of difference between travelling and standing waves.

	Travelling Waves	Standing Waves
1.	Particular conditions of the medium namely crests and troughs or compressions and rarefactions appear to travel with a definite speed depending on density and elasticity (or tension) of the medium.	Segments of the medium between two points called nodes appear to contract and dialate. Each particle or element of the medium vibrates to and fro like a pendulum.
2.	The amplitude of vibration of all the particles is the same.	At nodes the amplitude is zero but at antinodes the amplitude is maximum.
3.	All the particles pass through their mean positions with maximum velocity one after the other.	At nodes the particle velocity is zero and at antinodes it is maximum.
4.	Energy is transferred from particle to particle with a definite speed.	The energy surges back and forth in a segment but does not move past a point.
5.	In a travelling wave the particles attain their maximum velocity one after the other.	In a stationary wave the maximum velocity is different at different points It is zero at nodes but maximum at antinodes. But all the particles attain their respective maximum velocity simultaneously.
6.	In a travelling wave each region is subjected to equal strains one after the other.	In case of standing waves strain is maximum at nodes and zero at antinodes.
7.	There is no point where there is no change of density.	Antinodes are points of no change of density but at nodes there is maximum change of density.

INTEXT QUESTIONS 23.7

1. *Does energy flow across a point in case of stationary waves? Give a reason for your answer?*

   ....

2. *What is the distance between two successive nodes, and what is the distance between a node and next antinode?*

   ....

3. *What is meant by the statement that pressure nodes are displacement antinodes and pressure antinodes are displacement nodes.*

   ....

4. *Stationary waves of frequency 170 Hz are formed in air. If the velocity of the waves is 340 ms^{-1}, what is the shortest distance between (i) two nodes (ii) two antinode (iii) node and an antinode?*

....

23.8 CHARACTERISTICS OF MUSICAL SOUND

The following are the characteristics of musical sounds by mean of which we can distinguish one musical sound from another.

(i) Pitch and frequency, (ii) Loudness and Intensity (iii) Quality or Timbre.

23.8.1 Pitch and Frequency

The term *pitch is the characteristic of musical notes that enables us to classify a note as 'high' or 'low'.* It is a subjective quantity which cannot be measured by an instrument. It depends upon an objective quantity the frequency. However, there exists no one-to-one correspondence between the two. A shrill, sharp or acute sound is said to be of high pitch. But a dull, grave and flat note is said to be of low pitch. Roaring of lion though of high intensity is of low pitch. On the other hand the buzzing of mosquito though of low intensity is of high pitch.

23.8.2 Intensity and Loudness

The *intensity of waves is the average amount of energy transported by the wave per unit area per second normally across a surface at the given point.* There is a large range of intensities over which the ear is sensitive. As such logarithmic scale rather than arithmatic intensity scale is more convenient. Hence, the intensity level β of a sound wave is defined by the equation.

$$\beta = 10 \log I / I_0 \qquad (23.26)$$

Where I_0 is arbitrarily chosen reference intensity taken as 10^{-12} Wm^{-2}. This value corresponds to faintest sound that can be heard. Intensity level is expressed in decibels, abbreviated db. If the intensity of a sound wave equals to I_0 or 10^{-12} Wm^{-2}, its intensity level is then $I_0 = 0$ db. Within the range of audibility, sensitivity of ear varies with the frequency. ***The threshold audibility at any frequency is the minimum intensity of sound at that frequency which can be detected.***

The ***loudness*** is subjective measure of intensity for it depends upon the response of the ear of the listener. The standard of perceived loudness is the ***sone***. A sone is the loudness experienced by a listener with normal hearing when 1kilo hertz tone of intensity 40db is presented to both ears.

The range of frequencies and intensities to which ear is sensitive have been represented in a diagram in Fig. 23.14 which is in fact a graph between frequency in hertz versus intensity level I in decibels. This is a graph of auditory area of good hearing. The following points may be noted:

1. The lower part of the curve shows that the ear is most sensitive for frequencies between 2000 to 3000 Hz. Where the threshold of hearing is about 5 db. Threshold of hearing in general is zero decibel.
2. At intensities above those corresponding to the upper part of the curve, the sensation changes from one of hearing to discomfort and even pain. This curve represents the threshold of feeling.
3. Loudness increases with intensity, but there is no definite relation between the two.
4. Pure tones of same intensity but different frequencies do not necessarily produce equal loudness.
5. The height of the upper curve is constant at a level of 120 db for all frequencies.

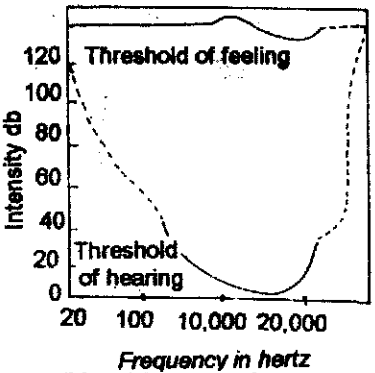


Fig. 23.14: *Auditory area between threshold of hearing and threshold of feeling*

The following are the factors on which the intensity of sound waves depends

1. ***Amplitude of vibration*** : $I\ \alpha\ a^2$ where a is amplitude of vibration of the wave.
2. ***Distance between the observer and the Source*** : $I \alpha 1/r^2$ where r is the distance of the observer from the source (provided it is a point source).
3. ***Intensity is inversely proportional to the square of frequency of the wave*** ν $\left(I\ \alpha\ \nu^2\right)$
4. ***Intensity is directly proportional to the density of the medium*** $(I\ \alpha\ \rho)$.

23.8.3 Quality

It is the *characteristic of sound waves which enables us to distinguish between two notes of the same pitch and intensity but sounded by two different instruments.* No instrument except a tuning fork can emit a pure note, that is a note of one particular frequency, In general, when a note of frequency n is sounded, in addition to it, notes of hihger frequences $2n$, $3n$, $4n$, ... may be produced. These notes, in general, may have different amplitudes and phase relation. The resultant wave form of the waves emitted determine the quality of the note emitted. Quality, like loudness and pitch is a subjective quantity. It depend upon the resultant wave form which is a subjective quantity.

INTEXT QUESTIONS 23.8

1. *How is the intensity level of sound defined?*

   ....

2. *What is meant by threshold audibility at a given frequency?*

   ....

3. *How pitch is related to frequency?*

   ....

4. *What is the unit in which intensity level is usually expressed? How is it related to watt* m^{-2}*?*

   ....

5. *What is that characteristic of musical sounds which enables you distinguish between two notes of the same frequency, and same intensity but sounded by two different instruments?*

   ....

6. *How is that you are able to recognise the voice of a near and dear?*

   ....

23.9 ORGAN PIPES

It is the simplest form of a wind instrument. A wooden or metal pipe producing musical sound is known as organ pipe. Flute is an example of organ pipe. If both the ends of the pipe are open we call it an ***open pipe***. However, if the lower end is closed, we call it a ***closed pipe***. When we blow in gently, almost a pure tone is heard. This pure tone is called a ***fundamental note***. But, when we blow harder, we also hear notes of frequencies which are integral multiple of the frequency of the fundamental note. These frequencies are called **overtones.**

23.9.1 Overtones of Organ Pipes

Before discussing the overtones of organ pipes let us bear in mind the following:

(i) At the closed end of a pipe there can be no motion of the air particles. Hence the closed end must be node.

(ii) At the open end of the pipe, the change in density must be zero since this end is in communication with atmosphere. Further, since the strain is zero, hence this end must be an antinode.

(iii) Between two antinodes there must be a node and between two nodes there must be an antinode.

(iv) The distance between two successive nodes or successive antinodes is $\lambda/2$ and that between a node an next antinode, the distance $\lambda/4$.

(a) Open pipe: The simplest mode of vibrations of the air column is shown in Fig.23.15 (a). At each of the two ends there is an antinode and between the two antinodes there is a node. Since the distance between a node and next antinode is $\lambda/4$, therefore, the length l of the pipe is $l = \lambda/4 + \lambda/4 = \lambda/2$ or $\lambda = 2l$

The frequency of the note produced is

$$n_1 = \upsilon/\lambda = \upsilon/2l$$

The next mode of vibration of the air column is shown in Fig.23.15. One more node and one more antinode has been produced. In this case

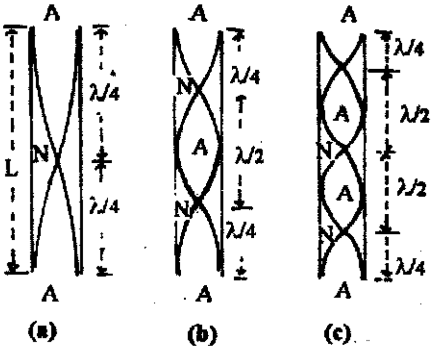


Fig. 23.15: *Harmonisc of an open organ pipe. The curves represent the wave of the longitudinal standing waves*

$$l = \lambda/4 + \lambda/4 + \lambda/4 + \lambda/4 = \lambda$$

The frequency of the note is

$$n_2 = \upsilon/\lambda = \upsilon/l = 2\upsilon/2l$$

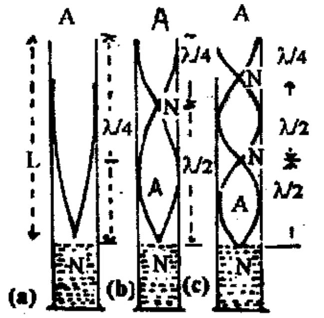
That is $n_2 = 2n_1$

The note produced is called second harmonic or 1st ***overtone***. To get the second harmonic you have to blow harder. But if you blow still harder one more node and one more antinode is produced [Fig. 23.15 (b)]. Thus, in this case

$$l = \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{4} \text{ or } \lambda = \frac{2l}{3}$$

Therefore, the frequency of the note emitted is

$$n_3 = \frac{\upsilon}{\lambda} = \frac{3\upsilon}{2l} = 3n^3$$



Fundamental or First Harmonic Third Marmonic Fifth Marmonic

Fig. 23.16: *Harmonics of a closed organ pipe. The curves represent wave form of the longitudinal standing waves.*

The note produced is called the 3rd harmonic or 2nd overtone.

(b) Closed pipe : The simplest manner in which the air column can vibrate in a closed pipe is shown in Fig. 23.16 (a). There is an antinode at the open end and a node at the closed end. The wave length of the wave produced is given by

$$l = \lambda/4 \text{ or } \lambda = 4l$$

Therefore, the frequency of the note emitted is

$$n_1 = \upsilon/\lambda = \upsilon/4l$$

The note produced is called *fundamental* or first harmonic. On blowing harder one more node and more antinode will be produced (Fig. 23.16b). The wave length of the note produced is given by

$$l = \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{3\lambda}{4} \text{ or } \lambda = \frac{4l}{3}$$

The frequency of the note emitted will be

$$n_3 = \frac{v}{\lambda} = \frac{3v}{4l} = 3\,n_1$$

The note produced is called the first overtone or the 3rd harmonic. On blowing still harder one more node and one more antinode will be produced (Fig.23.16). The wavelength of the note produced is then given by

$$l = \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{5\lambda}{4} \text{ or } \lambda = \frac{4l}{5}$$

The frequency of the note emitted then will be

$$n_3 = \frac{v}{\lambda} = \frac{5v}{4l} = 5\,n_1$$

The note produced is called the second overtone or the 5th harmonic.

On comparison with the notes emitted by the open and closed pipe, you will find that the open pipe is richer in overtones. In closed pipe, the even order harmonics are missing.

INTEXT QUESTIONS 23.9

1. *Out of open and closed organ pipes which one is richer in overtone?*

   ....

2. *What is the ratio of the frequencies of the notes emitted (i) by an open pipe and (ii) by a closed pipe*

   ....

3. *What will be the effect of temperature, if any, on the frequency of the fundamental note of an open pipe?*

   ....

4. *Show that the frequency of the fundamental note of open end pipe is two times the frequency of the fundamental note of a closed end pipe of the same length.*

   ....

23.10 NOISE POLLUTION

When the sensation of sound changes from one of hearing to discomfort or even pain such a sound is a noise and if it persits for a long time, it has harmful effects on certain organ of human beings. Noise is also one of the by-products of industrialisation and misuse of modern amenities provided by science to human beings. We have already defined decible, the unit of measuring intensity level. We summarise here under the sources or description of noises and their effects as perceived by the human beings.

Table 23.1 : *Sources of Noise and their Effects*

Source	**Intensity Level in decibels**	**Perceived Effect by human being**
Threshold of hearing	0 (=10^{-12} Wm^{-2})	Just audible
Rustle of leaves	10	Quiet
Average whisper	20	Quiet
Radio at low volume	40	Quiet
Quiet automobile	50	moderately loud
Ordinary conversation	65	do
Busy street traffic	70 to 80	loud
Motor bike and heavy vehicles	90	very loud
Jet engine about 35 m away	105	Uncomfortable
Lightening	120 (=1 Wm^{-2})	do
Jet plane at take off	150	Painful sound

(a) Effect of Noise Pollution

1. It causes impairment of hearing. Prolong exposure of noise at 85 or more than 85 db causes severe damage to the inner part of the ear.
2. It increases the rate of heart beat and causes dilation of the pupil of eye.
3. It causes emotional disturbance, anxiety and nervousness.
4. It causes severe headache leading to vomiting.

(b) Methods of Reducing Noise Pollution

1. Shifting of old industries and setting new ones away from the dwellings.
2. Better maintenance of machinery, regular oiling and lubrication of moving parts.
3. Better design of engines and machines.
4. Restriction on use of loudspeakers and amplifiers.
5. Restricting the use of fire crackers, bands and loud speakers during

religious, political and marriage processions.

6. Planting trees on roads for intercepting the path of sound.
7. Intercepting the path of sound by sound absorbing materials.
8. Using muffs and cotton plugs.

(c) Shock Waves

When a source of waves is travelling faster than the sound waves, shock waves are produced. The familiar example is the explosive sound heared by an observer when a supersonic plane flies past over the head of the observer. It may be pointed out that the object which moves with a speed greater than the speed of sound is itself a source of sound.

INTEXT QUESTIONS 23.10

1. *What is the effect of Noise Pollution?*

   ....

2. *State briefly the chief sources of noise.*

   ....

3. *What could be the various methods of reducing noise?*

   ....

4. *Describe briefly how a bow shock wave is produced.*

   ....

23.11 THE DOPPLER EFFECT

While waiting on a railway platform for the arrival of a train you would have observed that the pitch of the whistle when the engine approaches you and when the engine moves away from you changes . You will note that the pitch is higher when the engine approaches but is lower when the engine moves away from you.

Apparent change of frequency observed owing to the relative motion of the observer and the source is known as Doppler effect and the principle which explains it is called the Doppler Principle.

Let υ = velocity of the sound waves relative to the medium, that is air;

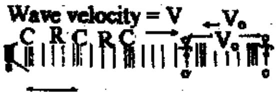
υ_s = velocity of the source; and

υ_0 = velocity of the observer.

It is important to note that the wave originated at a moving source does not affect the speed of the sound. The speed υ is the property of the medium. The wave forgets the source as it leaves the source. Let us suppose that the source, the observer and the sound waves travel from left to right. Let us first consider the ***effect of motion of the source.*** A particular note which leaves the source at a given time after one second arrives at the point A such that SA = υ. In this time, the source moves a distance υ_s. Hence all the n waves that the source has emitted in 1 second are contained in the space $s'x = \upsilon - \upsilon_s$. Thus length of each wave has decreased to

$$\lambda' = \frac{\upsilon - \upsilon_s}{n}$$

Now let us consider the ***effect of motion of the observer.*** A particular wave which arrives at 0 at a particular time after one second will be at B such that OB = v. But in the mean time, the observer moves from O to O'. Hence only the waves contained in the space O'B have passed across the observer in 1s. The number of the waves passing across the observer in 1s is therefore,



λ- Distance between two successive compression C.

Fig. 23.17: *The effect of motion of the observer is to reduce the number of waves passing across the observer.*

$$n' = (\upsilon - \upsilon_o)/\lambda' \tag{23.28}$$

Substituting for λ' from eq. (23.27) we get

$$n' = \frac{\upsilon - \upsilon_0}{\upsilon - \upsilon_s}\, n \tag{23.29}$$

Where n' is the observed frequency when both observer and source are moving in the direction from the source to the observer.

In using the above equation that is eq. (23.29) ***take v the velocity of sound in the direction from the source to the observer as positive;*** and υ_0 and υ_s as positive if these are in the direction of υ and negative if in the direction opposite to the direction of υ.

The following examples will help to understand the application of Doppler's principle in better way.

Example 23.1 : *The light from a given star on spectroscopic analysis shows a shift towards the red end of the spectrum of a spectral line of an element. If this shift, called the red shift, is 0.032%, find the velocity of recession of the star.*

Solution : In this case the source is source of light waves situated in the star. The observer is at rest on the Earth. We have shown that in such a case

$$\lambda' = \frac{\upsilon - \upsilon_s}{n}$$

But $\quad \upsilon = n\lambda$ or $n = \upsilon/\lambda$

Therefore, $$\lambda' = \frac{\upsilon - \upsilon_s}{\upsilon/\lambda} = \lambda\,\frac{(\upsilon - \upsilon_s)}{\upsilon}$$

or $$\lambda' = \lambda\left(1 - \frac{\upsilon_s}{\upsilon}\right)$$

or $$\frac{\lambda' - \lambda}{\lambda} = -\frac{v_s}{v}$$

or $$\frac{\Delta\lambda}{\lambda} = -\frac{v_s}{v}$$

In the present case $\frac{\Delta\lambda}{\lambda} = 0.032/100$; and $v = c = 3 \times 10^8$ m s^{-1}

Therefore $v_s = -v \quad = -3 \times 10^8 \times 0.032/100 = -9.6 \times 10^4$ m s^{-1}

Example 23.2 : *A radar sends waves of frequency 8.1 x 10^9 Hz towards an aeroplane. The reflected wave from the aeroplane shows a frequency shift of 2.7 x 10^4 Hz on the higher side on the radar screen. Find the velocity of the aeroplane in the line of sight.*

Solution : In the present case the aeroplane acts as a mirror for radio-waves. The image of the source of radio-waves owing to reflection from the surface of the aeroplane, is formed as far behind the reflecting surface as the source in front of the reflecting surface. In the present case this image acts as the source and the radar screen as the observer. The velocity of the image is two times the velocity of the plane.

If x be the velocity of the plane then

$$v_s = 2x;\ v_0 = 0,\ v = c = 3 \times 10^8\ \text{ms}^{-1};\ n = 8.1 \times 10^9\text{Hz}.$$
$$\Delta n = n' - n = 2.7 \times 10^4\text{Hz}$$

Since, $v_0 = 0$, therefore, from eq. (23.29) $n' = (v - v_0) / (v - v_s)\, n$, we get

$$n' = \frac{v - 0}{v - v_s} n$$

or $$n' = \left[\frac{1}{1 - v_s / v}\right] n \quad \text{or} \quad n' = \left[1 - \frac{v_s}{v}\right] n \quad \text{or} \quad n' - n = \frac{v_s}{v} n$$

$$\Delta n = (-v_s/v)n' = -v_s/v(n + \Delta n)$$

$$\therefore \Delta n = \frac{v_s}{v} n = \frac{2x}{v} n$$

$$\therefore\ x = \frac{1}{2}\,\frac{\Delta n}{n}\, v$$

$$= \frac{1}{2} \times \frac{2.7 \times 10^4}{8.1 \times 10^9} \times 3 \times 10^8$$

$$= 0.5 \times 10^3\text{ms}^{-1}$$

INTEXT QUESTIONS 23.12

1. *A SONAR system fixed in a submarine operates at frequency 40.0 kHz. An enemy submarine moves towards the SONAR system with a speed of 100 m s^{-1}. What is the frequency of the sound reflected by the sonar. Take the speed of sound in water to be 1450 ms^{-1}.*

   [Hints : the image formed by the submarine acts as a source moving with a v_s = 200 ms^{-1} towards the SONAR. The SONAR acts as an observer at rest that is $v_0 = 0$ use the relation $n' = (v - v_o)\, n/(v - v_s)$ you will get $n' = 1450 \times 40/(1450 - 200)$, which gives n' = 46.4 kHz.

   ....

2. *An engine, blowing a whistle of frequency 200 Hz moves with a velocity 16 ms^{-1} towards a hill from which a well defined echo is heard. Calculate the frequency of the echo as heard by the driver. Velocity of sound in air is 340 ms^{-1}.*

   [Hints : relation to be used $n' = (v - v_o) \times n/(v - v_s)$ when approaching the hill $n' = v \times n/(v - v_s)$.

   ....

23.13 TERMINAL QUESTIONS

1. How will you define a wave in the most general form?
2. Explain using a suitable mechanical model, the propagation of (i) transverse waves (ii) longitudinal wave. Define the term wavelength and frequency.
3. Define angular frequency ω and propagation constant k and hence show that the velocity of the wave propagation is $v = \omega/k = \eta\lambda$
4. Derive the equation of a simple harmonic wave of angular frequency ω and propagation constant k propagating along ox.
5. What are the essential properties of the medium for propagation of (i) transverse waves (ii) longitudinal waves
6. Develop an expression for the intensity of the wave in terms of density of the medium, velocity of the wave, the amplitude of the wave and the frequency of the wave
7. Write Newton's formula for the velocity of sound in a gas and explain Laplace's correction.
8. When do two waves interfere (i) constructively (ii) destructively?
9. Show using trigonometry that when two simple harmonic waves of the same angular frequency ω and same wavelength λ but of amplitudes a_1 and a_2 are superposed, the resultant amplitude is $A = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$ where ϕ is the phase difference between them. What would be the value of A, for $\phi = 0$ (ii) for $\phi = 2\pi$ (iii) for $\phi = (2m + 1)\pi$?

10. What are beats? How are they formed? Explain graphically.

11. Discuss graphically the formation of stationary waves. Why are these wave called stationary waves? Define nodes and antinodes.

12. State the points of difference between stationary and travelling waves.

13. Derive the equation of a stationary wave and show that displacement nodes are pressure antinodes and displacement antinodes are pressure nodes?

14. What are the characteristics of musical sounds. Explain.

15. What is a decibel (symbol db)? What is meant by 'threshold of hearing' and 'threshold of feeling'?

16. What is meant by quality of sound? Explain with examples?

17. Discuss the harmonics of organ pipes. Show that an open pipe is richer in harmonics.

18. Show that (i) the frequency of the fundamental note of an open pipe is two times the frequency of the fundamental note of a closed pipe of same length (ii) to produce a fundamental note of same frequency, the length of the open pipe must be two times the length of the closed pipe.

19. Describe an experiment to demonstrate existence of nodes and antinodes in an organ pipes?

20. Briefly state the causes of noise pollution, its harmful effects and methods of minimising it.

21. Explain the Doppler's effect and derive the Doppler's equation. How does this equation get modified if the medium in which the sound travels is also moving.

22. Discuss the application of Doppler's effect in (i) measuring the velocity of recession of stars, (ii) velocity of enemy plane by RADAR and (iii) velocity of enemy boat by SONAR?

23. For a plane wave in air of frequency 1000 Hz and amplitude 2 x 10^{-8} m, calculate the intensity. Take = 1.3 kg m^{-3} and wave velocity 340 ms^{-1}.

24. Calculate the velocity of sound in a gas in which two waves of wave lengths 1.00 m and 1.01 m produce 10 beats in 3 seconds [Hints : $\upsilon/1.00 - \upsilon/1.01 = 10/3$, Therefore $\upsilon = 336.67$ ms^{-1}].

25. What will be the length of a closed pipe if the lowest note has a frequency 256 Hz at 20°C. Velocity of sound at 0°C = 332 ms^{-1}.

26. The frequency of the sound waves emitted by a source is 1 kHz calculate the wavelength of the waves as perceived by the observer when (a) the source is stationary (b) the source is moving with a velocity of 50 m s^{-1} towards the observer, (c) the source is moving with a velocity of 50 ms^{-1} away from the observer. Velocity of sound in air is 350 ms^{-1}.

23.14 WHAT YOU HAVE LEARNT

- When a particular condition observed at a point in space at a particular time reappear at a certain other point in space after a lapse of certain time without the transfer of particles of the medium, we say that a wave is travelling.
- The distance between two nearest points in a wave motion vibrating in the same phase is called wavelength.
- The equation of a simple harmonic wave propagating along ox is $y = a \sin(\omega t - kx)$
- The energy transmitted per second across a unit area normal to the area is called intensity of sound.
- If the vibrations are perpendicular to the direction of propagation it is said to be transverse wave and when the vibrations are parallel to the direction of propagation then it is said to be longitudinal wave. Velocity of transverse wave pulse is $v = \sqrt{T/m}$ and that of longitudinal wave is $v = \sqrt{E/\rho}$
- On reflection from a denser medium phase is reversed by π. But there is no phase reversal on reflection from a rarer medium.
- When two waves are superposed, the resultant displacement at each point is equal to the sum or difference in the displacements of the two waves at that point. It is known as superposition principle.
- Superposition of two waves of the same wavelength same frequency and same amplitude travelling in the opposite directions with the same speed results in a wave whose wave form does not move. Such a wave is called a standing wave or a stationary wave.
- In a stationary wave at certain points the displacement amplitude is zero but strain is maximum, such points are called nodes. But at certain other points the displacement amplitude is maximum but strain is zero, these points are called antinodes.
- The distance between a node and next antinode is $\lambda/4$ the distance between two successive nodes or successive antinode is $\lambda/2$. It is, therefore, obvious that between two nodes, there is an antinode and between two antinodes there is a node.
- Intensity level is defined by the equation $\beta = 10 \log I/I_0$ where I_0 is an arbitrarily chosen reference intensity taken 10^{-12} W m^{-2}. Intensity level is expressed in decibels (Symbol. db)
- Threshold of hearing is the minimum intensity of sound of a given frequency which can be just detected.
- Quality of a note is the characteristic of musical sounds which enable us to distinguish two notes of the same pitch and same loudness but sounded by two different instruments.

- These are notes of higher frequency each being an integral multiple of the lowest frequency. They are called overtones.

CHECK YOUR ANSWERS

Intext Questions 23.1

1. See section 23.3.4
2. If p be the path difference, then the phase difference is $\phi = \frac{2\pi}{\lambda} p$
3. ϕ
4. Intensity is the energy that flows per second normally across a unit area. At a given point, depends upon square of amplitude, square of frequency.

Intext Questions 23.2

1. Newton assumed that compression and rarefaction caused by sound waves takes place under isothermal condition.
3. Newton assumed that isothemal conditions instead of adiabatic conditions for sound propagation.
4. 357°C

Intext Questions 23.3

1. $v = \sqrt{\frac{T}{m}}$
2. Therefore, $n = \frac{l}{\lambda} \sqrt{T/m}$

Further, for the simplest mode of vibration, at the two ends of the string, there are nodes and in between the two nodes is an antinode. Therefore, $l = \lambda/2$ or $\lambda = 2l$, hence, $n = 1/2l \sqrt{T/m}$. If the string vibrates in p segments, then $l = p\,\lambda/2$ or $\lambda = 2l/p$. Then $n = p/2l \sqrt{T/m}$

Intext Questions 23.4

For answers to all questions see text.

Intext Questions 23.5

For answers refer to text.

Intext Questions 23.6

1. 25/9; 2. Beats with frequency 4Hz are produced
3. Frequency of beats is $\Delta\gamma$
4 517, on loading the frequency of A decreases from 517 to 507.

Intext Questions 23.7

1. No, energy swings back and forth in a segment..

2. Distance between two successive nodes is $\lambda/2$,and between a node and antinode is $\lambda/4$

4. (i) 1 m, (ii) 1m, (iii) 1/4 m.

5. See text.

6. Wave form which is the resultant of a number of harmonic determines the quality and hence enables to recognise the voice of known persons.

Intext Questions 23.8

4. For a closed pipe in case of fundamental note $l = \lambda/4$ or $\lambda = 4l$, therefore $n = v/\lambda = v/4l$ (i)

For an open pipe $l' = \lambda/2$. Therefore $n' = v/2l$ (ii)

Comparing (i) and (ii) we find that $n' = 2n$ when $l = l'$.

For other questions see text.