

# 3

## LAWS OF MOTION

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### 3.1 INTRODUCTION

In the previous lesson you have studied that it is possible to describe the motion of an object in terms of its displacement, velocity and acceleration. But have you ever wondered what causes an object to move? What causes a ball rolling along the ground to come to a stop, apparently on its own?

From our everyday experience we know that we need to push or pull an almirah if we wish to change its position in a room. Similarly, a football has to be kicked in order to send it over a large distance. A cricket ball has to be hit hard with a bat to send it across the boundary for a six. Some kind of muscular activity is involved in these situations and the action and its effect is quite visible.

There are, however, many situations where the action is not visible. For example, what makes rain drops fall on ground? What makes the earth go round the sun? In this lesson you will discover a close relation between force and motion in the form of Newton's laws of motion. The concept of force developed in this lesson will be useful in different branches of physics. Let us study about Newton's laws of motion which enable us to predict the behaviour of a particle or a system of particles under the influence of different forces.

### 3.2 OBJECTIVES

After studying this lesson, you should be able to,

- *explain the meaning of inertia and relate it to force;*
  - *state Newton's laws of motion and illustrate them with examples;*
  - *define momentum, impulse and calculate them in a given situation;*
  - *explain the law of conservation of momentum and illustrate it with examples;*
  - *define coefficient of friction and distinguish between static friction, kinetic friction and rolling friction;*
  - *suggest different methods of reducing friction and highlight the role of friction in every-day life; and*
  - *analyse a given situation and apply Newton's laws of motion using free body diagrams.*
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### 3.3 CONCEPT OF FORCE AND INERTIA

A large number of objects around us are known to remain wherever they are placed. These objects cannot move on their own from one place to another place. These objects have to be forced to change their state of rest. Similarly an object which is moving at constant velocity has to be forced to change its state of uniform motion. ***The tendency of an object to remain in its state of rest or in its state of uniform linear motion is called inertia.***

The state of rest or the state of motion of an object are not absolute. In the previous lesson you have already studied that an object at rest with respect to one observer may appear to be in motion with respect to some other observer. Observations show that ***the change in velocity of an object can only be brought if some net force acts on the body.***

You are very well familiar with the term force. We use it in so many situations in our everyday life. We are exerting force when we are pulling pushing, kicking, hitting etc. Though a force is not visible but its effect can be seen and experienced. Forces are known to have two kinds of effects :

- (a) ***They may change the shape and the size of an object.*** For example, a balloon changes shape according to the forces acting on it.
- (b) ***Forces also influence the motion of an object.*** A force can set an object into motion or it can bring a moving object to rest. A force can also change the direction of motion.

In lesson seven you will study about another effect of force - the rotational or turning effect of force.

#### 3.3.1 Force and Change in Motion

Motion of any body is characterised by its velocity. We come across many situations where the velocity of an object is gradually increasing or decreasing. For example, in the case of a body falling freely, the velocity of the object increases continuously. Similarly, in the case of a ball rolled on some horizontal surface, the velocity of the ball is seen to decrease gradually to zero.

Observations show that some force is responsible for change in velocity of a body. ***Direction of velocity of a body is always in the direction of its motion. The velocity of the body will change depending upon the direction of the force acting on it.*** If some force acts on a body in the direction of its motion, the velocity of the body will increase in magnitude. If the direction of force on the body is opposite to the direction of motion the magnitude of velocity will decrease. In both these cases the object moves in a straight line. However, if some force acts on a body in a direction perpendicular to its velocity, the magnitude of velocity of the body remains constant. Such a force is able to change only the direction of velocity of the body. It is important to note that ***velocity of a body changes as long as some force is acting on it.***

Force is a vector quantity, for this reason when several forces act on a body simultaneously, their single equivalent force can be found by vector addition about which you have already read in lesson 1. Every force has a

magnitude and a direction. The effect, a force acting on a system can produce, depends on,

- (a) the magnitude and direction of the force;
- (b) the point of application of force; and
- (c) the duration for which it acts.

### 3.3.2 Newton's First Law of Motion

We see that in order to move a trolley at constant velocity it has to be continuously pushed or pulled. Similarly, a horse is seen pulling the cart moving at constant velocity. Is there any net force acting on the trolley or the cart in the situations mentioned here?

Galileo was the first to state that in the absence of any external influence a body cannot only be at rest but also moves uniformly in a straight line. He, therefore, regarded uniform rectilinear motion also to be natural state of bodies.

It is now known that action of the horse on the cart is needed for balancing out the force of friction on the cart. Similarly force of friction on the trolley can be overcome by continuously pushing or pulling it. Galileo drew the conclusion that in the absence of force of friction, a body such as a cart or a trolley, when once set in motion would continue to move at constant velocity.

Isaac Newton generalised Galileo's conclusions in the form of a law known as *Newton's first law of motion* which states that *a body continues to be in the state of rest or in the state of uniform motion in a straight line unless and until it is acted upon by some net external force.*

As you know, the state of rest or motion of a body depends on its relative position with respect to the observer. A person in a running car is at rest with respect to another person in the same car. But the same person is in motion with respect to a person standing on the road. For this reason it becomes necessary to record measurements of changes in position, velocity and acceleration and force with respect to a chosen frame of reference.

A reference frame relative to which a body in translatory motion has constant velocity if no net external force acts on it is known as an ***inertial frame of reference***. Such a name follows from the property of inertia of bodies due to which they preserve their state of rest or state of uniform linear motion. A reference frame fixed to the earth (for all practical purposes) is considered an inertial frame of reference.

Now take a break and try to solve the following questions.

### INTEXT QUESTIONS 3.1

1. Is it correct to state that a body always moves in the direction of net external force acting on it  
.....
2. What physical quantity is a measure of inertia of a body?  
.....

3. Can a force change only the direction of velocity of an object keeping its magnitude constant?  
.....
4. What are the different effects a force is capable of producing?  
.....

### 3.4 CONCEPT OF MOMENTUM

Study of collision of bodies has revealed that there is a quantity mass multiplied by velocity whose net value for the colliding bodies remains unchanged in a collision. **The product of mass  $m$  of a body and its velocity  $v$  is called its linear momentum or simply momentum  $p$ .** So

$$p = mv \quad (3.1)$$

In the SI system of units momentum is measured in kg m/s. Momentum is a vector quantity. The direction of momentum vector is the same as the direction of velocity vector. Momentum of an object, therefore, can change on account of change in its magnitude alone or direction alone or both. The following examples illustrate this point.

**Example 3.1 :** A 2 kg object is allowed to fall freely at  $t = 0$  s. What will be its momentum at (a)  $t = 0$  s, (b)  $t = 1$  s and (c)  $t = 2$  s during its free-fall ?

**Solution :** (a) As velocity of the object at  $t = 0$  s is zero, the momentum of the object will also be zero.

(b) At  $t = 1$  s, the velocity of the object will be 9.8 m/s [use  $v = v_0 + at$ ] pointing downward. So the momentum of the object will be

$$p_1 = (2 \text{ kg}) \times (9.8 \text{ m/s}) = 19.6 \text{ kg m/s pointing downward.}$$

(c) At  $t = 2$  s, the velocity of the object will be 19.6 m/s pointing downward. So the momentum of the object will now be

$$p_2 = (2 \text{ kg}) \times (19.6 \text{ m/s}) = 39.2 \text{ kg m/s pointing downward.}$$

Thus we see that the momentum of a freely-falling body increases in magnitude only and points in the same direction. Now you think what causes the momentum of a freely-falling body to change in magnitude?

**Example 3.2 :** A rubber ball of mass 0.2 kg strikes a rigid vertical wall with a speed of 10 m/s and rebounds along the original path with the same speed. Find the change in momentum of the ball.

**Solution :** Here the momentum of the ball has same magnitude before and after the impact but there is a reversal in its direction. The magnitude of momentum is (0.2 kg) (10 m/s) i.e. 2 kg m/s.

If we consider initial momentum vector to be along +x axis, the final momentum vector will be along -x axis. So if  $p_i = 2 \text{ kg m/s}$ ,  $p_f = -2 \text{ kg m/s}$  and change in momentum of the ball =  $p_f - p_i = (-2 \text{ kg m/s}) - (2 \text{ kg m/s}) = -4 \text{ kg m/s}$ .

Here negative sign shows that the momentum of the ball changes by 4 kg m/s in the direction of -x axis. What causes this change in momentum of the ball?

In actual practice a rubber ball rebounds from a rigid wall with a speed which is less than its speed before the impact. In such a case the magnitude of the momentum also change.

### 3.4.1 Newton's Second Law of Motion

It is now known that a body moving at constant velocity will have constant momentum. It is already known from Newton's first law of motion that no net external force acts on such a body.

In example 3.1 we have seen that the momentum of a ball falling freely under gravity increases with time. It is well known that such a body falls under the action of gravitational force acting on it. So there appears to be a connection between change in momentum of an object, net force acting on it and the time for which it is acting. **Newton's second law of motion** gives a quantitative relation between these. According to it **the rate of change of momentum of a body is directly proportional to the net force acting on the body. Change in momentum of the body takes place in the direction of net external force acting on the body.**

This means that if  $\Delta p$  is the change in momentum of a body in time  $\Delta t$  due to some net external force  $F_{\text{net}}$  on it, then

$$F_{\text{net}} \propto \frac{\Delta p}{\Delta t}$$

$$\text{or } F_{\text{net}} = k \frac{\Delta p}{\Delta t}$$

Here value of  $k$ , the constant of proportionality, depends on the choice of units for  $F$ ,  $p$  and  $t$ .

In the SI system of units  $p$  is measured in kg m/s,  $t$  is in s and  $F$  is measured in kg m/s<sup>2</sup>. That gives  $k = 1$ . So

$$F_{\text{net}} = \frac{\Delta p}{\Delta t} \quad (3.1)$$

In the limit  $\Delta t \rightarrow 0$ , we can write

$$\boxed{F_{\text{net}} = \frac{dp}{dt}} \quad (3.2)$$

From equation 3.1, it follows that change in momentum of a body is in the direction of net external force on it. Change in momentum of a body during any time is taken as final momentum of the body minus initial momentum of the body.

From Newton's first law of motion it follows that a body at rest can not move on its own. Similarly if it is already moving, it can not change its velocity on its own. You may recall that if a body has constant velocity, its acceleration must be zero. You may also recall that by change in velocity we mean change in magnitude of velocity or change in its direction or both.

Newton's first law also tells us that some net force must act on an object to change its state of motion. So net force acting on an object is the cause of acceleration of the object. Newton's second law of motion gives a quantitative relation between force and acceleration. If due to the application of  $F_{\text{net}}$  force, the velocity of body changes by  $dv$  in time  $dt$ , then  $dp = m dv$ .

Then the equation 3.2 gives,

$$F_{\text{net}} = m \frac{dv}{dt}$$

$$F_{\text{net}} = ma$$

(3.3)

**Example 3.3:** A ball of mass 0.4 kg which starts rolling on the ground at 20 m/s when pushed comes to a stop after 10 seconds. Calculate the force which stops the ball, assuming it to be constant in magnitude throughout.

**Solution :** Given,

$$m = 0.4 \text{ kg} \quad u = 20 \text{ m/s}$$

$$v = 0 \text{ m/s} \quad t = 10 \text{ s}$$

$$\begin{aligned} \text{So } F &= \frac{m(v - u)}{t} = \frac{0.4 \text{ kg} (0 \text{ m/s} - 20 \text{ m/s})}{10 \text{ s}} \\ &= -0.8 \text{ kg m/s}^2 = -0.8 \text{ N} \end{aligned}$$

Here negative sign shows that force on the ball is opposite to its direction of motion. It is evident from the fact that there is a decrease in the momentum of the ball. You are already aware that the ball comes to rest because of the action of force of friction on it.

**Impulse :** From equation 3.1 it follows that it is the product  $F \Delta t$  that determines change in momentum of a body. So a desired change in momentum can be brought about by a large force acting for a small duration or by a small force acting for a long duration of time. The quantity  $F \Delta t$  is called the impulse. Thus, **the change in momentum of a body is equal to the impulse.**

Like momentum, impulse is also a vector quantity and is in the direction of the change in momentum. Its SI unit of measurement is kg m/s or Ns.

In deriving equation 3.1 it was assumed that force  $F$  remains constant over time  $\Delta t$  of its action. There are many situation where the time of action of force is very small, for example, during the impact of a ball and a bat. In such cases  $F$  is considered as the average force.

**Inertial Mass :** The mass of a body defined by Newton's second law of motion is called inertial mass of the body. From Newton's second law

$$m = \frac{F}{a}$$

The known value of force and the experimental determination of the acceleration of a body as a result of the force can help us to measure inertial mass of the body.

Now stop and by solving the following questions check how much you have learnt.

### INTEXT QUESTIONS 3.2

- Two objects of different masses have same momentum. Which of them is moving faster?

2. A ball is thrown up at a speed of 20 m/s by a thrower. If the ball returns to the thrower with the same speed of 20 m/s only, will there be any change in
  - (a) momentum of the ball? .....
  - (b) magnitude of the momentum of the ball? .....
3. When a ball falls from a height, its momentum increases? What causes increase in its momentum?  
.....
4. In which case will there be larger change in momentum of the object?
  - (a) A 150 N force acts for 0.1 s on a 2 kg object initially at rest.
  - (b) A 100 N force acts for 0.25 on a 4 kg. object initially at rest.
5. Is it correct to say that a fast moving object possesses more force than a slow moving object? Why?  
.....
6. An object is moving at a constant speed in a circular path. Does the object have constant momentum? Why?  
.....

**Example 3.4 :** A constant force of 50 N is applied to a body of 10 kg moving initially with a speed of 10 m/s. How long will it take the body to stop if the force acts on it in a direction opposite to its velocity.

**Solution :** Given,

- Mass of the body,  $m = 10 \text{ kg}$   
 $F_{\text{net}} = -50 \text{ N}$   
 $v_0 = 10 \text{ m/s}$   
 $v = 0$

To find :  $t = ?$

$$\text{Formula(s)} \quad F_{\text{net}} = m \left( \frac{v - v_0}{t} \right) \quad \text{or} \quad -50 \text{ N} = 10 \text{ kg} \left( \frac{0 - 10 \text{ m/s}}{t} \right)$$

$$\text{or } t = \frac{-100 \text{ kg m/s}}{-50 \text{ N}} = \frac{100 \text{ kg m/s}}{50 \text{ kg m/s}^2}, \quad t = 2 \text{ second.}$$

### 3.5 FORCES IN PAIRS

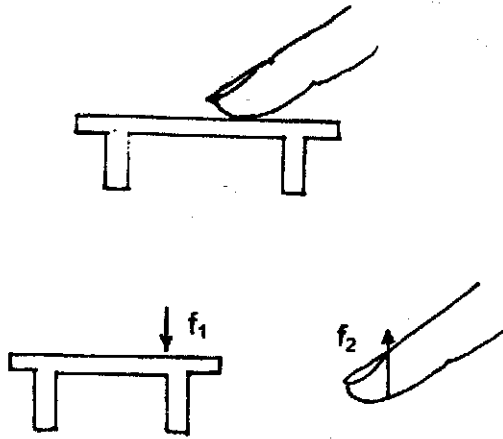
It is the gravitational pull of the earth which allows an object to accelerate toward earth. Does the object also pull the earth? Similarly when we push an almirah, does the almirah also push us? If so, why don't we move in the direction of that force? These situations compel us to ask whether a single force such as a push or a pull exists? It has been observed that actions of two bodies on each other are always mutual. Here, by 'action' we mean 'force of interaction'. So, whenever two bodies interact, they exert force on each other. The force of interaction could be action-at-a-distance type or a contact force type. Thus forces have been found to emerge in pairs always. But generally we are concerned with one of the forces in a pair which is acting on the object of interest.

#### 3.5.1 Newton's Third Law of Motion

On the basis of his study of interaction between bodies, Newton formulated *third law of motion* which states that *to every action there is*

**equal and opposite reaction. The action and reaction act on different bodies.**

Here by 'action' and 'reaction' we mean force. It follows from here that a single isolated force does not exist. Thus, when we exert some force on a table by pressing our finger against it, the table also exerts a force of equal magnitude on our finger in the upward direction as shown in Fig. 3.1. Do the forces  $f_1$  and  $f_2$  shown here cancel out? It is important to note that  $f_1$  and  $f_2$  are acting on different bodies.



**Fig 3.1 :** Force  $f_1$  exerted by the finger on table and force  $f_2$  exerted by the table on the finger.

The action and reaction in a given situation appear as a pair of forces. Any one of them cannot exist without the other.

If one goes by the literal meaning of words, reaction always follows an action. Whereas action and reaction introduced in Newton's third law exist simultaneously. For this reason it is better to state Newton's third law as **when two objects interact, the force exerted by the first object on the second (the action) is equal in magnitude and opposite in direction to the force by the second object on the first (reaction).**

Vectorially, if  $F_{12}$  is the force which object 1 experiences due to object 2 and  $F_{21}$  is the force which object 2 experiences due to object 1, then

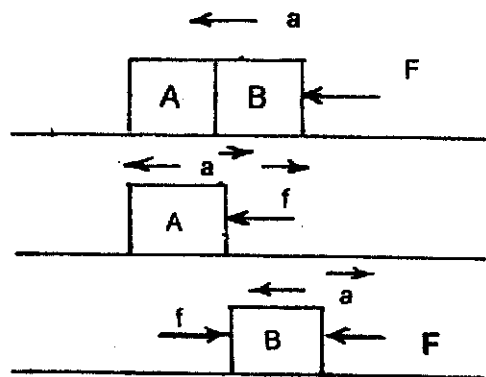
$$F_{12} = -F_{21} \quad (3.4)$$

If  $m_1$  and  $m_2$  are the masses of objects 1 and 2 respectively then

$$m_1 a_1 = -m_2 a_2$$

### 3.5.2 Internal and External Forces

As you have studied that some net external force on an object causes the object to accelerate. If a car breaks down it can either be towed by some other vehicle or pushed from behind. Here the force exerted on the car by the towing vehicle or the push from behind acts as the **external force** on the car. Can the car be moved by a person who is sitting in the car and pushing it from inside? Such a push is called **internal force** and has no role to play as far as motion of car is concerned. To make the distinction between internal forces and external forces clear, let us look at some specific examples.



**Fig 3.2:** Two blocks on a frictionless horizontal surface.

a) Consider two blocks A and B, placed in contact with each other on a frictionless horizontal surface. When some external force  $f$  is applied to

them in the direction shown in Fig. 3.2, both the blocks move to the left with the same acceleration. But what causes the block A on the left to accelerate? It is the force  $f$  which the block B on the right exerts on it. In accordance with Newton's third law, block on the left also exerts force  $f$  on the block on the right. However, the forces of magnitude  $f$  each which these blocks exert on each other are internal to the system of blocks.

b) Whenever bodies collide, the forces involved are 'internal' to the colliding bodies taken as a system. So the absence of any external force on the system demands that total momentum of the system should be conserved.

### 3.5.3 Forces in Equilibrium

**Number of forces acting on an object or a system are said to be in equilibrium if the vector sum of all the forces is zero.** In accordance with Newton's second law, acceleration of such an object or system will be zero. Let us take an example.

Consider a block of mass  $m$  resting on some horizontal surface as shown in Fig 3.3(a). In this situation we can talk about a number of forces. Such as,

- (i) gravitational pull of the earth *on the block* equal in magnitude to  $mg$ .
- (ii) force of magnitude  $mg$  which the block exerts *on earth*.

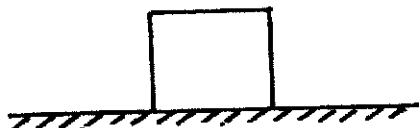


Fig. 3.3(a)

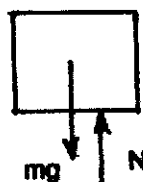


Fig. 3.3(b)

- (iii) force of magnitude  $mg$  which the block exerts *on the surface* normally on which it is resting.
- (iv) force of normal reaction  $N$  exerted by the surface *on the block* in accordance with Newton's third law.

Fig. 3.3(b) shows the forces acting on the block. Since the block is in equilibrium  $N = mg$ . In the situation described above, is there any frictional force acting on the block?

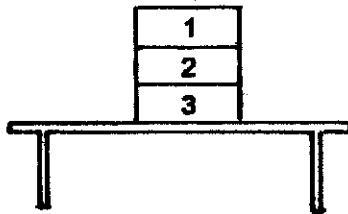
In the next section you will study problems where more than two forces are in equilibrium.

Now, it is time for you to check how much you have learnt.

### INTEXT QUESTIONS 3.3

1. When a high jumper leaves the ground, where does the force which accelerates the jumper upwards come from?  
.....
2. Identify action - reaction forces in each of the following situations:  
(a) A man kicks a football. ....

- (b) Earth pulls the moon. ....
- (c) A ball hits a wall. ....
3. What is the magnitude and direction of the gravitational force which a 60 kg woman exerts on earth?  
.....
4. Name the forces which are in equilibrium in each of the following situations?  
(a) A book resting on a table .....  
(b) A cork floating in water .....  
(c) A pendulum bob suspended from the ceiling with the help of a string.  
.....
5. Three blocks of mass  $m$  each are placed on top of a table as shown. Name a force each which is internal and external to the system of blocks.



- .....
6. "A woman exerts a large force on an almirah to push it forward. The woman is not pushed backward because the almirah exerts a small force on woman" Is the argument given here correct? Why?  
.....

### 3.6 CONSERVATION OF MOMENTUM

Momentum possesses a very useful property of being conserved under certain conditions. It has been experimentally confirmed and has been proved also that if two bodies interact, the vector sum of the momenta of these bodies remains unchanged provided the force of mutual interaction is the only force acting on the bodies. The same has been found to be true for more than two bodies interacting with each other. Generally, a number of bodies interacting with each other are said to be forming a system. If the bodies in a system do not interact with bodies outside the system, the system is said to be a closed system or an isolated system. The law of conservation of momentum is valid for isolated systems only and can be stated as ***the vector sum of the momenta of bodies comprising an isolated system remains constant.***

Here, it follows that it is the total momentum of the bodies forming an isolated system remains unchanged and the momentum of individual bodies may change in magnitude alone or direction alone or both depending upon the type of interaction. What causes the momentum of individual bodies in an isolated system to change momentum?

Conservation of linear momentum is applicable in wide range of phenomena such as collisions, explosions, nuclear reactions, radioactive decay etc.

### 3.6.1 Conservation of Momentum as a Consequence of Newton's Laws

According to *Newton's second law of motion*, equation 3.1, the change in momentum  $\Delta p$  of a body when a force  $F$  acts on it for time  $\Delta t$  is

$$\Delta p = F \Delta t.$$

It follows from here that if no force acts on the body, the change in momentum of the body will be zero. Therefore the momentum of the body will remain unchanged. This explanation can be applied to a system of bodies also.

*Newton's third law* can also be used to arrive at the same result. Consider an isolated system of two bodies A and B which interact with each other for time  $\Delta t$ . If  $F_A$  and  $F_B$  are the forces which they exert on each other then, in accordance with Newton's third law

$$F_A = -F_B$$

$$\text{or } \frac{\Delta p_A}{\Delta t} = -\frac{\Delta p_B}{\Delta t}$$

$$\text{or } \Delta p_A + \Delta p_B = 0 \text{ or } \Delta p_{\text{total}} = 0$$

So there is no change in the momentum of the system.

### 3.6.2 A Few Illustrations of the Conservation of Momentum

**a) Recoil of a gun :** When a bullet is fired from a gun, the gun recoils. The velocity  $v_2$  of the recoil of the gun can be found by using the law of conservation of momentum. Let  $m$  be the mass of the bullet being fired from a gun of mass  $M$ . If  $v_1$  is the velocity of the bullet, then the velocity  $v_2$  of the gun will be such that

$$m v_1 + M v_2 = 0$$

$$\text{or } m v_1 = -M v_2$$

$$\text{or } v_2 = \frac{-m}{M} v_1 \quad (3.5)$$

Here, negative sign shows that  $v_2$  is in opposite direction to  $v_1$ . As generally  $m \ll M$ , the recoil velocity of the gun is much smaller than the velocity of the bullet.

**b) Collision :** In a collision we may regard the colliding bodies as forming a system. In the absence of any external force on the colliding bodies such as the force of friction, the system can be considered to be an isolated system. The forces of interaction between the colliding bodies being internal to the system will not be able to change the momentum of the colliding bodies.

**Example 3.5 :** Two coupled trolleys, each of mass  $m$ , are moving with initial velocity  $v$ . They collide and couple with three stationary trolleys also of mass  $m$  each. What will be the velocity of the trolleys after the impact?

**Solution :** Let  $v'$  be the velocity of the trolleys after the impact.

Momentum before collision =  $2 mv$

Momentum after collision =  $5 mv'$

In accordance with the law of conservation of momentum

$$2 mv = 5 mv'$$

$$\text{or } v' = \frac{2}{5}v$$

**c) Explosion of a bomb :** A bomb explodes into fragments through the release of stored energy. Consider a bomb at rest initially which explodes into two fragments A and B. As the momentum of the bomb was zero before explosion, the total momentum of the two fragments formed will also be zero after the explosion. For this reason, the two fragments will fly off in opposite direction with equal momenta. If the masses of the two fragments are equal, the velocities of the two fragments will also be equal in magnitude.

**d) Rocket propulsion :** Flight of a rocket is an important practical application of conservation of momentum. A rocket is a two-body system. It consists of a shell with a fuel tank, which can be considered as one body. The fuel in the fuel tank can be considered as the other body. The shell is provided with a nozzle through which high pressure gases are made to escape. On firing the rocket, the combustion of the fuel produces gases at very high pressure and temperature. Due to their high pressure, these gases escape from nozzle at a high velocity. The rocket flies in the opposite direction. The propulsion of the rocket can be explained as follows:

For an earth based observer, the total momentum of the rocket (shell and fuel) is zero before launching. As a result of combustion of fuel, the ejected gases acquire certain momentum. Considering the shell and the fuel to be forming a closed system, their total momentum must also be zero after launching. Therefore, the shell acquires a momentum equal in magnitude to the momentum of the ejected gases but in opposite direction.

Propulsion of a rocket is different from the recoil of a gun in atleast one important aspect as far as the conservation of momentum is concerned. Unlike the bullet, the entire gas from the rocket is not ejected at once. In actual practice, the rocket loses fuel at some rate which is generally constant.

Again it is time to check your progress. Solve the following questions.

### INTEXT QUESTIONS 3.4

1. Is the momentum of a ball falling freely conserved? Why?  
.....
2. Can a bomb, initially at rest, explode into  
(a) two pieces which fly in opposite directions to each other?  
(b) three pieces which fly in mutually perpendicular directions?  
.....
3. Why is the velocity of recoil of a gun much smaller than the velocity of the bullet?  
.....
4. Two balls of mass 0.2 kg each are moving along the same line towards each other with a velocity of 10 m/s just before colliding. What will be the total momentum of the balls after the collision?  
.....

### 3.7 FRICTION

It is well known that when we push a ball to make it roll along the ground, the ball does not continue to travel for ever. It rather comes to rest after travelling some distance. Thus the momentum of the ball which was imparted to it during initial push changes to zero. We have already studied that some force on the ball is responsible for change in its momentum. Such a force called the **frictional force** emerges whenever bodies in contact tend to move or move with respect to each other. It is the force of friction on an almirah which has to be overcome when we push or pull it along the floor to change its place.

**Force of friction is a contact force and is always directed along the surfaces in contact.** It is commonly known that friction is caused by roughness of the surfaces in contact. For this reason deliberate attempts are made to make the surfaces rough or smooth depending upon the requirement.

Friction opposes the motion of objects, causes wear and tear and is responsible for loss of mechanical energy. But then, it is due to friction only that we are able to walk, drive vehicles and stop moving vehicles. Friction thus plays a dual role in our lives.

#### 3.7.1 Static and Kinetic Friction

It is common experience that certain minimum force is required to move an object over a surface. To illustrate this point let us consider a block resting on some horizontal surface as shown in Fig. 3.4. Let some external force  $F_{\text{ext}}$  be applied to the block in the direction shown, such that the block does not move. This is possible only if some other force is acting on the block which is of the same magnitude as  $F_{\text{ext}}$  but is in opposite direction. Such a force is called the force of **static friction** and is represented by symbol  $f_s$ . As  $F_{\text{ext}}$  is increased,  $f_s$  also increases and remains equal to  $F_{\text{ext}}$  in magnitude until it reaches a critical value  $f_s(\text{max})$ . When  $F_{\text{ext}}$  is increased further, the block starts to slide and is then subject to **kinetic friction**. It is common experience that force needed to set an object into motion is larger than the force needed to keep it moving at constant velocity. **For this reason the maximum force of static friction  $f_s(\text{max})$  between a pair of surfaces in contact will be larger than the force of kinetic friction  $f_k$  between them.** Fig. 3.5 shows the variation of the force of friction with the external force.

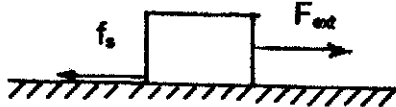


Fig 3.4: Forces acting on the block.

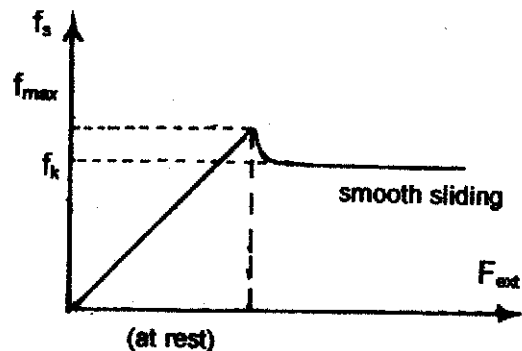


Fig 3.5: Variation of force of friction.

For a given pair of surfaces in contact, what are the **factors** on which  $f_s(\text{max})$  and  $f_k$  depend? It is an experimental fact that  $f_s(\text{max})$  is directly proportional to the normal force  $F_N$ . So

$$f_s(\text{max}) \propto F_N \text{ or } f_s(\text{max}) = \mu_s F_N \quad (3.6)$$

Here  $\mu_s$  is called the **coefficient of static friction**.

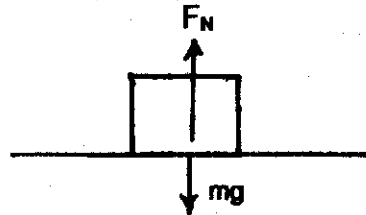
$$\text{As } f_s \leq f_s(\text{max}), f_s \leq \mu_s F_N$$

It has also been experimentally found that **maximum force of static friction between a pair of surfaces is independent of the area of contact**.

$$\text{Similarly, } f_k = \mu_k F_N \quad (3.7)$$

Where  $\mu_k$  is the **coefficient of kinetic friction**. Generally  $\mu_s > \mu_k$ , but there are exceptions. Moreover, coefficients  $\mu_s$  and  $\mu_k$  are not really constants for any pair of surface such as wood on wood or rubber on concrete etc. **Value of  $\mu_s$  and  $\mu_k$  for a given pair of materials depends on roughness, cleanness, temperature, humidity etc.**

As discussed earlier, the normal force  $F_N$  of the surface on the block can be found by knowing the force with which the block presses the surface. As shown in Fig 3.6, the normal force  $F_N$  on the block will be  $mg$  where  $m$  is the mass of the block.



**Fig 3.6:** Normal force on the block.

**Example 3.6:** A 2 kg block is resting on a horizontal surface. The coefficient of static friction between the surfaces in contact is 0.25. Find the maximum force of static friction between the surfaces in contact.

**Solution:**

$$\text{Here } F_N = mg$$

$$\text{and } \mu_s = 0.25$$

$$\begin{aligned} \text{As } f_s(\text{max}) &= \mu_s F_N = \mu_s mg \\ &= (0.25) (2 \text{ kg}) (9.8 \text{ m/s}^2) \\ &= 4.9 \text{ N.} \end{aligned}$$

**Example 3.7:** A 5 kg block is on a horizontal surface for which  $\mu_k = 0.1$ . What will be the acceleration of the block if it is pulled by a 10 N force acting on it in the horizontal direction?

**Solution:**

$$\text{As } f_k = \mu_k F_N \text{ where } F_N = mg$$

$$\begin{aligned} \text{Therefore } f_k &= \mu_k mg \\ &= (0.1) (5 \text{ kg}) (9.8 \text{ m/s}^2) \\ &= 4.9 \text{ kg m/s}^2 = 4.9 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Net force on the block} &= F_{\text{ext}} - f_k \\ &= 10 \text{ N} - 4.9 \text{ N} \\ &= 5.1 \text{ N} \end{aligned}$$

$$\text{and therefore, acceleration} = a = \frac{F_{\text{net}}}{m} = \frac{5.1 \text{ N}}{5 \text{ kg}} = 1.02 \text{ m/s}^2$$

So the block will have an acceleration of  $1.02 \text{ m/s}^2$  in the direction of externally applied force.

### 3.7.2 Rolling Friction

It is common experience that it is easier to push or pull objects which are on wheels. The motion of a wheel is different from the sliding motion. It is rolling motion. The friction in the case of rolling motion is known as **rolling friction**. For the same normal force, rolling friction is much smaller than sliding friction. For example when steel wheels roll over steel rails, rolling friction is about  $1/100^{\text{th}}$  of sliding friction between steel and steel. Typical values for coefficient of rolling friction  $\mu_r$  are 0.006 for steel on steel and 0.02 – 0.04 for rubber on concrete.



**Fig 3.7:** Deformation of the flat surface during rolling.

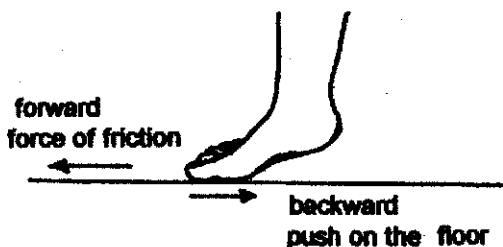
In the case of rolling motion of a wheel on some surface, the point of contact of the wheel has zero relative velocity with respect to the surface. This means the point of contact of the wheel has no sliding motion. Only the centre of the wheel moves forward. The origin of rolling friction can be explained as follows. When a ball rolls on some flat surface, it slightly deforms the surface. Figure 3.7 shows the deformation of the flat surface during rolling. Because of the surface deformations, a rolling ball has to climb a hill as long as it is rolling. If the surfaces in contact are harder, lesser will be the surface deformations and hence smaller the rolling friction.

### 3.7.3 Importance of Friction

Friction plays very important role in our life. It is because of friction only that we are able to walk or hold a tooth brush in our hand.

Appreciable friction is required between the brake shoe and the rim of a bicycle for the purpose of stopping it when required. And interestingly, it is the force of friction only which sets a vehicle driven by an engine into motion. Discussed below is a situation highlighting role of friction.

**Walking :** When we are standing on the floor there is no net force acting on us in the horizontal or vertical direction. As we begin to walk, we must have some acceleration in the forward direction. Which means that some net external force must act on us in the forward direction. From where does this force come? We create this force on us by pushing the floor in the backward direction. It is due to the friction between the floor and our feet that we are able to obtain the desired force on us. Fig 3.8 shows the direction in which floor is to be pushed by the feet to experience force of friction on it in the forward direction. That explain, why it is difficult for us to walk on a smooth/slipper surface.



**Fig 3.8:** Forces on the feet while walking.

Force of friction offers **hindrance** too. It is the force of friction which brings a moving bicycle to rest on a level road if we stop paddling it. Do we need to paddle a bicycle to move it at constant speed on a level road? Friction is responsible for great amount of energy loss in machines. As you will study in lesson 6, work is required to be done in overcoming force of friction. For this reason deliberate attempts are made to reduce friction where it is undesirable.

### 3.7.4 Methods of Reducing Friction

Wheel is considered to be greatest invention of mankind for the simple reason that rolling is much - much easier than sliding. Making use of rolling friction are the **ball bearings**. In a ball-bearing steel balls are placed between two co-axial cylinders as shown in Fig. 3.9. Generally one of the two cylinders is allowed to turn with respect to the other. Here the rotation of the balls is almost a frictionless motion. Ball-bearings find application in almost all type of vehicles and in electric motors such as electric fan.

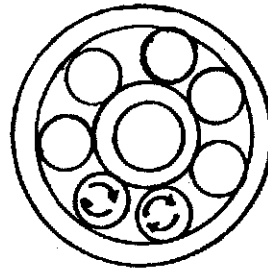


Fig 3.9: Balls in the ball-bearing.

Use of **lubricants** such as grease or oil between the surfaces in contact reduces friction considerably. In heavy machines, oil is made to flow over moving parts. It reduces frictional force between moving parts and also prevents them from getting over heated.

Presence of lubricants such as oil and grease in fact, changes the nature of friction from dry friction to fluid friction which is considerably smaller than the former.

**Flow of compressed and purified air** between the surfaces in contact also reduces friction. It also prevents dust and dirt from collecting on the moving parts. A hover craft is made to run on a cushion of air provided by powerful air pumps.

### INTEXT QUESTIONS 3.5

1. A table is lying on a floor. Is some force of friction acting on the table?  
.....
2. A book is lying on an inclined plane. Is some force of friction acting on the book?  
.....
3. With  $F_1 = 20\text{ N}$  and  $F_2 = 15\text{ N}$  the block of mass  $3\text{ kg}$  is sliding at constant speed. What will happen when  $F_2$  is removed?  
.....
4. In pushing a box up an inclined plane, is it better to push horizontally or to push parallel to the inclined plane? Why?  
.....

On the next page few Guidelines are given which will help you in solving problems: A number of solved examples have also been given at the end of this section. You may try them yourself before going through their solution.

## THE FREE BODY DIAGRAM TECHNIQUE

1. Draw a simple, neat diagram of the *system* as per the given description.
2. Isolate the *object of interest*. This object will be called the *Free Body* now.
3. Consider all *external forces* acting on the free body and mark them by arrows touching the free body with their line of action clearly represented.
4. Now apply Newton's second law  $\Sigma F = ma$   
(or  $\Sigma F_x = ma_x$  and  $\Sigma F_y = ma_y$ )

**Remember :** (i) There must be net force on the object along the direction of acceleration of the object. (ii) For obtaining a complete solution, you must have as many independent equations as the number of unknowns.

5. If the free body diagram of an object is made with reference to a non-inertial frame, pseudoforces like the centrifugal force will also be considered as real forces acting on the object in addition to all other external forces. The direction of such a force will be opposite to the direction of acceleration of the frame of reference.
6. In some situations you may have to consider rotational motion of the object and hence the expressions, torque,  $T = I\alpha$ , torque = force  $\times$  force arm are to be used. In such situations one has to balance clockwise moments' with anticlockwise moments about some axis about which the object rotates or tends to rotate.

**Example 3.8:** Two blocks of masses  $m_1$  and  $m_2$  are connected by a string and placed on a smooth horizontal surface. The block of mass  $m_2$  is pulled by a force  $F$  acting parallel to the horizontal surface. What will be the acceleration of the blocks and the tension in the string connecting the two blocks (assuming it to be horizontal)?

**Solution:** Let  $a$  be the acceleration of the blocks in the direction of  $F$  and let the tension in the string be  $T$ . On applying  $\Sigma F = ma$  in the component form to the free body diagram of  $m_1$  and  $m_2$  we get

$$N - (m_1 + m_2)g = 0 \quad \text{and} \quad F = (m_1 + m_2)a$$

$$N = (m_1 + m_2)g, \quad \text{and} \quad a = \frac{F}{m_1 + m_2}$$

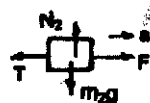
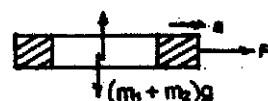
On applying  $\Sigma F = ma$  in the component form to the free body diagram of  $m_1$  we get

$$N_1 - m_1g = 0 \quad \text{and} \quad T = m_1 a$$

$$N_1 = m_1g \quad \text{and} \quad T = m_1 \left( \frac{F}{m_1 + m_2} \right)$$

$$\text{or} \quad T = \left( \frac{m_1}{m_1 + m_2} \right) F$$

Apply  $\Sigma F = ma$  once again to the free body diagram of  $m_2$  and see whether you get the same expressions for  $a$  and  $T$ .



**Example 3.9:** Two masses  $m_1$  and  $m_2$  ( $m_1 > m_2$ ) are connected at the two ends of a light inextensible string that passes over a light frictionless fixed pulley. Find the acceleration of the masses and the tension in the string connecting them when the masses are released.

**Solution:** Let  $a$  be the acceleration of mass  $m_1$  downward. The acceleration of mass  $m_2$  will also be  $a$  only but upward. (Why?). Let  $T$  be the tension in the string connecting the two masses.

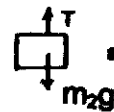
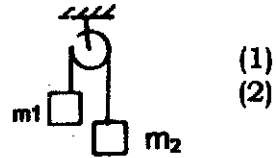
On applying  $\Sigma F = ma$  to  $m_1$  and  $m_2$  we get

$$m_1 g - T = m_1 a \quad (1)$$

$$T - m_2 g = m_2 a \quad (2)$$

On solving equations (1) and (2) for  $a$  and  $T$  we get

$$a = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) g \quad T = \left( \frac{2m_1 m_2}{m_1 + m_2} \right) g$$



At this stage you can check the prediction of the results thus obtained for the extreme values of the variables (i.e.  $m_1$  and  $m_2$ ). Either take  $m_1 = m_2$  or  $m_1 \gg m_2$  and see whether  $a$  and  $T$  take values as expected.

**Example 3.10:** A trolley of mass  $M = 10$  kg is connected to a block of mass  $m = 2$  kg with the help of a massless inextensible string passing over a light frictionless pulley as shown in Fig (a). The coefficient of kinetic friction between the trolley and the surface is  $\mu_k = 0.02$ . Find

- acceleration of the trolley.
- tension in the string.

**Solution:** Fig (b), (c) shows the free body diagrams of the trolley and the block. Let  $a$  be the acceleration of the block and the trolley.

For the trolley,  $F_N = Mg$  and  $T - f_k = Ma$  where  $f_k = \mu_k F_N = \mu_k Mg$

$$\text{So } T - \mu_k Mg = Ma \quad (1)$$

$$\text{For the block } mg - T = ma \quad (2)$$

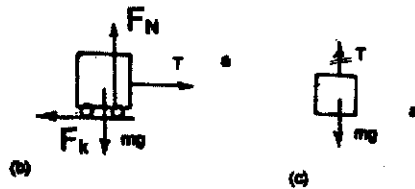
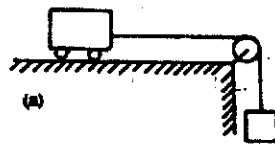
On adding equations (1) and (2) we get  $mg - \mu_k Mg = (M + m)a$

$$\begin{aligned} \text{or } a &= \frac{mg - \mu_k Mg}{M + m} = \frac{(2\text{kg})(9.8\text{ m/s}^2) - (0.02)(10\text{kg})(9.8\text{ m/s}^2)}{(10\text{kg} + 2\text{kg})} \\ &= \frac{19.6\text{ kg m/s}^2 - 1.96\text{ kg m/s}^2}{12\text{kg}} = 1.47\text{ m/s}^2 \end{aligned}$$

$$\text{So } a = 1.47\text{ m/s}^2$$

$$\begin{aligned} \text{From equation (2) } T &= mg - ma = m(g - a) \\ &= 2\text{ kg}(9.8\text{ m/s}^2 - 1.47\text{ m/s}^2) \\ &= 2\text{ kg}(8.33\text{ m/s}^2) \end{aligned}$$

$$\text{So } T = 16.66\text{ N}$$



### INTEXT QUESTIONS 3.6

1. A block of mass  $m$  is held on a rough inclined surface of inclination  $\theta$ . Show in a diagram, various forces acting on the block.  
.....
2. A force of 100 N acts on two blocks A and B of masses 2 kg and 3 kg respectively placed in contact on a smooth horizontal surface as shown. What is the magnitude of force which block A exerts on block B?  
.....
3. What will be the tension in the string when a 5 kg object suspended from it is pulled up with  
(a) a velocity of 2 m/s ?  
(b) an acceleration of 2 m/s<sup>2</sup> ?  
.....
4. In the reference frame attached to a freely falling body of mass 2kg, what is the magnitude and direction of inertial force on the body?  
.....

### 3.8 WHAT YOU HAVE LEARNT

- The *inertia* of a body is its tendency to resist any change in its velocity.
- *Newton's first law* states that a body remains in a state of rest or in a state of uniform motion in a straight line as long as net external force acting on it is zero.
- For a single particle of mass  $m$  moving with velocity  $v$  we define a vector quantity  $p$  called the linear momentum as  $p = mv$ .
- The impulse of a force  $F$  on a particle is equal to the change in momentum of the body and is given by the area under the Force-time curve.
- *Newton's second law* states that the time rate of change of momentum of a body is proportional to the resultant force acting on the body.
- According to Newton's second law, acceleration produced in a body of constant mass is directly proportional to net external force acting on the body or  $F = ma$
- The proportionality factor between the net external force and the acceleration it causes in any object is the object's inertial mass.
- *Newton's third law* states that if two bodies A and B interact with each other, then the force which body A exerts on body B will be equal and opposite to the force which body B exerts on body A.
- According to the law of conservation of momentum if no net external force acts on a system of particles, the total momentum of the system of particles will remain constant regardless of the nature of forces between the particles.
- Frictional force is the force which acts on a body when the body attempts to slide, slides or rolls along a surface. The force of friction is always parallel to the surfaces in contact and opposite to the direction of motion of the object.
- In the case of objects driven by some internal mechanism such as a wheel driven by an engine, the force of friction causes motion.
- The maximum force of static friction  $f_s$  (max) between a body and a surface is proportional to the normal force  $N$  acting on the body. This maximum force occurs when the body is on the verge of sliding.
- For a body sliding on some surface, the magnitude of the force of kinetic friction  $f_k$  is given by  $f_k = \mu_k N$  where  $\mu_k$  is the coefficient of kinetic friction for the surfaces in contact.

- Use of rollers and ball - bearings reduces friction and associated energy losses considerably as rolling friction is much smaller than kinetic friction.
- Newton's laws of motion are applicable only in an inertial frame of reference. An inertial frame is one in which an isolated object has zero acceleration.
- For an object to be in static equilibrium, the vector sum of all the forces acting on it must be zero. This is a necessary and sufficient conditions for point objects only.

### 3.9 TERMINAL QUESTIONS

1. Which of the following will always be in the direction of net external force acting on the body?
 

(a) displacement	(b) velocity
(c) acceleration	(d) momentum.
2. When a constant net external force acts on an object, which of the following may not change?
 

(a) position	(b) speed	(c) velocity	(d) acceleration
--------------	-----------	--------------	------------------

 Justify your answer with an example each.
3. A 0.5 kg ball is dropped from such a height that it takes 4 seconds to reach the ground. Calculate the change in momentum of the ball.
4. In which case will there be larger change in momentum of a 2 kg object?
 

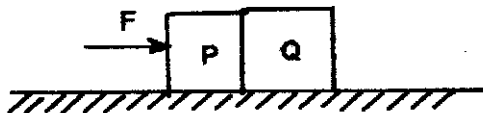
(a) When 10 N force acts on it for 1 s	(b) When 10 N force acts on it for 1 m.
--	---

 Calculate change in momentum in each case.
5. A ball of mass 0.2 kg falls in air with an acceleration of  $6 \text{ m/s}^2$ . Calculate the air drag on the ball.
6. A load of mass 20 kg is lifted with the help of a rope at constant acceleration. The load covers a height of 5 m in 2 seconds. Calculate the tension in the rope.
7. A ball of mass 0.1 kg moving at 10 m/s is deflected by a wall at the same speed in the direction shown. What is the magnitude of the change in momentum of the ball?
8. Find the average recoil force on a machine gun that is firing 150 bullets per minute, each with a speed of 900 m/s. Mass of each bullet is 12 g.
9. Explain why, when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest.
10. A constant force of magnitude 20 N acts on a body of mass 2 kg initially at rest for 2 seconds what will be the velocity of the object after
 

(a) 1 second from start?	(b) 3 seconds from start?
--------------------------	---------------------------
11. How does a force acting on a block in the direction shown here keep the block from sliding down the vertical wall?
12. A 2 kg block is resting on a horizontal surface. The coefficient of static friction between the block and the surface is 0.5. What will be the magnitude and direction of the force of friction on the block when the magnitude of the external force acting on the block in the direction shown is
 

(a) 0 N	(b) 4.9 N	(c) 9.8 N
---------	-----------	-----------
13. For a block on a surface the maximum force of static friction is 10 N. What will be the force of friction on the block when a 5 N external force is applied to it parallel to the surface on which it is resting?
14. What minimum force  $F$  is required to keep a 5 kg block at rest on an inclined plane of inclination  $30^\circ$ . The coefficient of static friction between the block and the inclined plane is 0.25.
15. Two blocks P and Q of masses  $m_1 = 2 \text{ kg}$  and  $m_2 = 3 \text{ kg}$  respectively are placed in contact with each other on a horizontal frictionless surface. Some external force  $F = 10\text{N}$  is applied to the block P in the direction shown in Fig Find the following.

- (a) acceleration of the blocks
- (b) force which the block P exerts on block Q.



16. Two blocks P and Q of masses  $m_1 = 2 \text{ kg}$  and  $m_2 = 4 \text{ kg}$  are connected to a third block R of mass  $M$  as shown in Fig. For what maximum value of  $M$  will the system be in equilibrium? The frictional force acting on each block is half the force of normal reaction on it.
17. Explain the role of friction in the case of bicycle brakes. What will happen if a few drops of oil are put on the rim?
18. A 2 kg. block is pushed up an incline plane of inclination  $\theta = 37^\circ$  imparting it a speed of 20 m/s. How much distance will the block travel before coming to rest? The coefficient of kinetic friction between the block and the incline is  $\mu_k = 0.5$ . Take  $g = 10 \text{ m/s}^2$  and use  $\sin 37^\circ = 0.6$ ,  $\cos 37^\circ = 0.8$ .

### 3.10 ANSWERS TO THE INTEXT QUESTIONS

#### Intext Questions 3.1

1. The statement is true for a body which was at rest before the application of some net force.
2. Inertial mass
3. Yes
4. A force can cause translational and rotational motion. It can also deform bodies.

#### Intext Questions 3.2

1. Object of smaller mass
2. (a) Yes (b) No.
3. Momentum of the falling ball increases because gravitational force acts on it in the direction of its motion.
4. In case (b) the change in momentum will be larger. It is the  $F \times \Delta t$  product that gives the change in momentum.
5. No. Force is not related with momentum. It is related with the rate of change of momentum.
6. No. Though the speed is constant, the velocity of the object changes due to change in direction. Hence its momentum will not be constant.

#### Intext Questions 3.3

1. The jumper is accelerated upward by the force which the ground exerts on the jumper. This force is the reaction to the force which the jumper exerts on the ground.

2. (a) If the force with which a man kicks a football is action then the force which the football exerts on the man will be its reaction.  
(b) If the force with which earth pulls the moon is action then the force which the moon exerts on the earth will be its reaction.  
(c) If the force which the ball exerts on the wall is the action then the force which the wall exerts on the ball will be its reaction.
3. Gravitational force on a 60 kg woman is  $60 \times 9.8 \text{ N}$  is 588 N. In accordance with Newton's third law the gravitational force exerted by the woman on the earth will also be 588 N. It will be directed radially outward.
4. (a) gravitational force on the book and the reaction force of the table.  
(b) gravitational force on the cork and the upthrust i.e. buoyant force.  
(c) gravitational force on the bob and the tension in the string connecting the bob.
5. Force internal to the system of blocks :  
The force with which block 2 presses block 3.  
Force external to the system of blocks:  
The reaction force of the table on the blocks.

**Intext Questions 3.4**

1. No. The momentum of the ball is not conserved because a freely - falling object is under the action of a net external force.
2. (a) Yes. The two pieces formed as a result of explosion will have equal and opposite momenta so that the total momentum of the pieces formed is zero as before the explosion.  
(b) Three momentum vectors can never produce a zero resultant if they are not in one plane, a bomb initially at rest cannot explode into three pieces which fly in mutually perpendicular directions. Such an occurrence is not possible as it violates the law of conservation of momentum.
3. In accordance with the law of conservation of momentum, a gun recoils with the same magnitude of momentum as that of the bullet. As the mass of the gun is generally much larger than the bullet, the recoil velocity of the gun is much smaller.
4. Zero.

**Intext Questions 3.5**

1. No, 2. Yes, 3.  $5 \text{ m/s}^2$
4. It is better to push the box by applying the force parallel to the inclined plane. When the force is applied horizontally, only a component of this force will act along the intended direction of motion. Moreover another component of this force will press the box on the incline thereby increasing the force of reaction of the incline on the box. As a result of this, the force of friction on the box will also increase.

**Intext Questions 3.6**

1. Various forces acting on the block are  
(a) force of reaction  $N$  of the inclined plane  
(b) force of friction  $f$   
(c) gravitational force  $mg$ .  
Fig. shows them in their appropriate directions.
2.  $60 \text{ N}$
3. (a)  $49 \text{ N}$  (b)  $59 \text{ N}$ .
4.  $19.6 \text{ N}$  in the vertically upward direction.