5

GRAVITATIONAL MOTION

5.1 INTRODUCTION

You may recall from your earlier studies that planets move around the sun in concentric circles with sun at the centre. You have also learnt in lesson 4 that a centripetal force is required for circular motion. What kind of force keeps the planets in their orbit? Throw a ball up or let a book slip from your hands. Where do they go? These fall on the Earth. It is a common observation that bodies close to the earth fall on it if they are free. Does the moon also fall towards the earth? Sir Issac Newton a, British Scientist, replied these questions through his law of gravitation.

In this lesson we shall study the law of gravitation, its universal nature and its consequences. This law has helped in the development of satellites, space probes and geostationary satellites. We shall also learn the difference between inertial and gravitational masses, satellite motion and phenomenon of weightlessness, in this lesson.

5.2 OBJECTIVES

After studying this lesson, you should be able to,

- *state and explain the universal law of gravitation;*
- *distinguish between inertial mass and gravitational mass and show their equivalence;*
- *analyse the variation in the value of 'g' due to different factors;*
- *state Kepler's laws of planetary motion;*
- *find relation between time of rotation and radius of orbit of a planet or satellite;*
- *identify the force responsible for planetary motion;*
- *calculate the orbital velocity and the escape velocity;*
- *recognise condition for a satellite to be geostationary and recognise the applications of satellites;* and
- *estimate the height for a synchronous satellite.*

5.3 UNIVERSAL LAW OF GRAVITATION

You see that a mango falls from the tree on the Earth. Does the Moon also fall towards the Earth? Yes indeed! [Had there been no centripetal force acting on the Moon, it would move away tangentially, see Fig. 5.1]

Is there any connection between these two falls? How motion of Moon about the Earth and motion of planets about the Sun are connected. Can you think of some relation between the stars and galaxy. Many more such questions about the position of heavenly bodies come to mind. Newton provided a coherent answer to these.

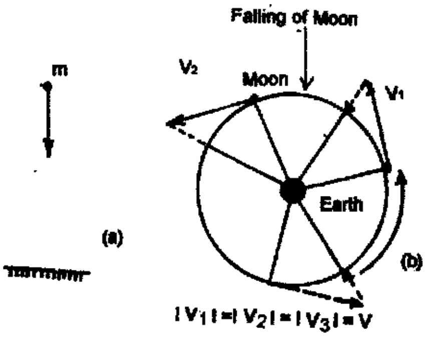


Fig 5.1 : *Falling of (a) Mass m and (b) of Moon towards Earth*

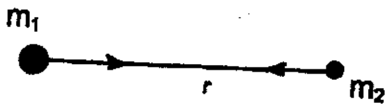


Fig 5.2 : *Gravitational attraction between masses m_1 and m_2 (m_1>m_2)*

Newton used the knowledge of Kepler's laws of planetary motion and his laws of motion to give the famous law of gravitation. The law is stated as ***Each particle in the universe attracts the other particle with a force which is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.***

The force is attractive only and acts along the line joining the masses (such a force is called ***central force***). Mathematically we may write

$$\boldsymbol{F}_G \ \alpha \ m_1 \times m_2$$

$$\boldsymbol{F}_G \ \alpha \frac{1}{r^2}$$

or $$\boxed{\boldsymbol{F}_G = -G\frac{m_1 m_2}{r^3}\hat{\boldsymbol{r}}} \qquad (5.1)$$

Here G, the constant of proportionality, is known as the ***universal constant of gravitation*** and $\boldsymbol{F}_G$ is the force of gravitation. The law may also be written as follows (See Fig 5.3).

$$\boldsymbol{F}_{12} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\boldsymbol{r}}_{21}$$

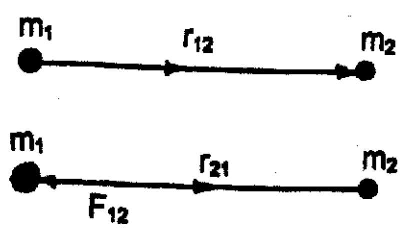


Fig 5.3 : *Gravitational attraction is mutual between m_1 and m_2.*

and $\boldsymbol{F}_{21} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$ (5.2)

It is evident from Fig 5.3 that $\boldsymbol{F}_{12}$ points in a direction opposite to $\boldsymbol{r}_2$ and it is the gravitational force exerted by m_2 on m_1 and $\boldsymbol{F}_{21}$ is the gravitational force on m_2 by mass m_1.

The ***gravitational force is always attractive***. Its magnitude is given by

$$|\boldsymbol{F}_G| = F_G = G\frac{m_1 m_2}{r^2} \quad (5.3)$$

Though this law is universal in nature, it cannot be used for atomic and subatomic particles. The force is mutual according to Newton's third law of motion that is m_1 attracts m_2 and in turn m_2 attracts m_1 so that

$$\boldsymbol{F}_{21} = -\boldsymbol{F}_{12} \quad (5.4)$$

The force of gravitation is very important for large masses. Let us see what happens at small distance in the following example.

Example 5.1 : *Two girls of masses 40 kg and 50 kg respectively are (a) standing at a distance of 0.50 m, (b) separated by a distance of 6.67 × 10⁶ m. Neglecting other forces, calculate the force of gravitational attraction between them. Given the value of gravitational constant $G = 6.67 \times 10^{-11}$ Nm² kg⁻².*

Solution: The force of gravitation is given by (in magnitude)

$$F_G = G\frac{m_1 m_2}{r^2}$$

In case (a)

$$F_G = \frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times 50\,\text{kg}\times 40\,\text{kg}}{(0.5)^2} = 5.336\times10^{-7}\,\text{N}$$

In case (b)

$$F_G = \frac{6.67\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}\times 50\,\text{kg}\times 40\,\text{kg}}{(6.67\times10^{6})^2\,\text{m}^2} = 3\times10^{-21}\,\text{N}$$

Note : All bodies are taken as point masses throughout this lesson.

5.3.1 Universal Constant of Gravitation

The constant G appearing in eqs. (5.1) to (5.3) is known as the universal constant of gravitation. Its value is

$G = 6.67 \times 10^{-11}$ Nm² kg⁻²

and its dimensions are $[G] = [M^{-1} L^3 T^{-2}]$

We can define G by taking $m_1 = m_2 = 1$ kg and $r = 1$ m in eq. (5.3).

Then $F_G = G$

Thus, ***the gravitational constant G is numerically equal to the force of attraction between two point masses 1 kg each placed at a distance of 1 m in air.***

The value of *G* is same for any pair of masses be it stars and galaxies or two shot-put spheres anywhere.

Example 5.2 : *A mass of 1 kg placed on the surface of the Earth is attracted by a force of 9.8 N. Taking mass of the Earth $M_E = 6.0 \times 10^{24}$ kg and its radius $R_E = 6.4 \times 10^6$ m, calculate the value of G.*

Solution: Using eq. (5.3) we can write

$$G = \frac{FR_E^2}{M_E\ m}$$

Substituting the values of given quantities, we get

$$G = \frac{9.8\text{N} \times (6.4 \times 10^6\ \text{m})^2}{6.0 \times 10^{24}\ \text{kg} \times 1\text{kg}} = 6.7 \times 10^{-11} \text{Nm}^2\ \text{kg}^{-2}$$

5.3.2 Universal Nature of the Law of Gravitation

You may be thinking that Newtons law of gravitation is applicable only for the members of the solar family. It is infact universal in nature. We can use it between any two mass points on the Earth or bodies in intergalactic space.

It was stated in the previous section that falling of the Moon and an object on the Earth have some connection. This may be agitating your mind - what connection? Let us explore it.

The distance of the moon from the Earth is $r_m = 3.84 \times 10^8$ m and the radius of the Earth is $R_E = 6.37 \times 10^6$ m.. It takes the Moon 27.3 days to go round the Earth once in its circular orbit. (Fig. 5.4). The centripetal acceleration of the Moon (being in circular orbit) is given by

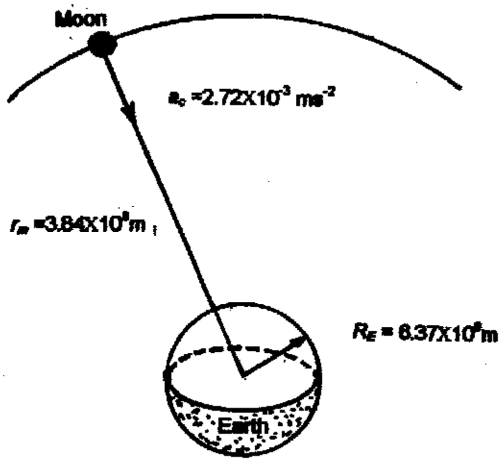


Fig 5.4 : *Moon revolving round the earth*

$$a_c = \frac{v^2}{r_m} = \frac{4\pi^2}{T^2} r_m \qquad \text{as } v = \frac{2\pi\, r_m}{T}$$

Substituting the values of T and r_m, we have,

$$a_c = \frac{4\pi^2 \times 3.84 \times 10^8\ m}{(27.3 \times 24 \times 3600)^2} = 2.72 \times 10^{-3} \text{ms}^{-2}$$

But the value of acceleration due to gravity acting on a freely falling body is $g = 9.8$ ms^{-2}.

Newton overcame the difficulty in the two values of acceleration by hypothesising that force of gravitation decreases with increase in distance from the Earth according to a definite law. Let us assume

$$\frac{g}{a_c} = \left(\frac{r_m}{R_E}\right)^m, \text{ which gives}$$

$$\frac{9.80\ \text{ms}^{-2}}{2.72 \times 10^{-3}\ \text{ms}^{-2}} = \left(\frac{3.84 \times 10^8 \text{m}}{6.37 \times 10^6 \text{m}}\right)^m$$

or $3600 = (60)^m$

Which gives $m = 2$, so that

$$\frac{g}{a_c} = \left(\frac{r_m}{R_E}\right)^2 \qquad (5.5)$$

$$\text{or } a_c \propto 1/r_m^{\,2} \qquad (5.6)$$

It shows that the acceleration (centripetal in case of Moon) produced by force of gravitation is inversely proportional to the square of the distance. It is directed towards the Earth. Similar conclusion may be drawn about the Sun-planet system. The force is directed towards the Sun. ***Thus gravitational force is an inverse square law central force.***

The law of gravitation applies not only to the Sun and planet system but also to any two point masses. It is universal in nature. The occurance of tides on sea, the formation of the universe, observation of moons of Jupiter and Saturn have been successfully explained with the help of the law of gravitation.

Let us take a pause and solve these questions.

INTEXT QUESTIONS 5.1

1. State the assumption, if any, made in the formulation of the law of gravitation.

   ....

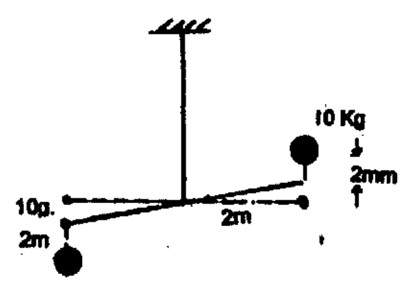
2. A mass m_1 is accelerated towards another mass m_2 such that $m_2 >> m_1$. Wny the acceleration of m_2 not seen towards m_1 ?

   ....

3. The force of gravitation between two bodies of equal masses, each m, placed at distance r is F_G. Find the magnitude of the force if (i) each body has mass 3 m and (ii) the separation is changed to $4r$.

   ....

4. In a cavendish experiment two lead spheres of masses 10 g each are suspended with a torsion rod of length 2 m. When two lead spheres of mass 10 kg each are brought near the suspended spheres as shown in the figure, the spheres are displaced through 2mm each. The torsional rod is deflected through an angle of 0.02 radian. Calculate the value of G if the torsional couple per unit twist of the suspension wire is 1.66×10^{-4} Nm.



   ....

5. At a point between the Earth and the Moon, the gravitational pulls of the Moon and the Earth are balanced. Estimate the distance of this point from the Earth.

   ....

5.4 INERTIAL MASS AND GRAVITATIONAL MASS

You may recall from Newton's second law we have

$$\mathbf{F} = m\mathbf{a}$$

and from Newton's law of Gravitation we have

$$\mathbf{F}_G = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

Does m stand for the same property in the two equations? It may be seen that the mass m characterises two different properties of an object, these are resistance to a change in its velocity and its gravitational interaction with other mass.

5.4.1 Inertial Mass

When we push a chair and a cot with the same force we find that chair moves faster than the cot. It shows that greater mass offers greater resistance to motion. Thus, mass characterises resistance to motion. This mass of the body is called the ***inertial mass*** m_i. Using Newton's second law of motion you have verified the law of conservation of linear momentum in lesson 3. According to this law

$$(m_1)_i \, \Delta \boldsymbol{v}_1 = -(m_2)_i \, \Delta \boldsymbol{v}_2 \qquad (5.7)$$

The masses $(m_1)_i$ and $(m_2)_i$ in this equation are inertial masses. If m_1 is a prototype kilogram, m_2 can be found by measuring changes in velocities. However, it is not easy to determine $\Delta \boldsymbol{v}_1$ and $\Delta \boldsymbol{v}_2$.

5.4.2 Gravitational Mass

In Newton's Law of gravitation you have seenthat the gravitational force F_G is proportional to the mass. Thus, this mass characterises the gravitational force and is known as the ***gravitational mass*** (m_G). To measure m_G we use the definition of weight,

$$\text{Thus, } \frac{W_1}{W_2} = \frac{(m_1)_G \, \boldsymbol{g}}{(m_2)_G \, \boldsymbol{g}} = \frac{(m_1)_G}{(m_2)_G} \qquad (5.8)$$

The weights W_1 and W_2 can be measured by a *pan or* spring balance. If $(m_1)_G$ is a prototype kilogram, $(m_2)_G$ can be found.

5.4.3 Concept of Weight

The ***force with which the Earth attracts a body is called its weight W***. Thus, weight is a force whose magnitude is proportional to the mass of the body and direction is downward towards the Earth. The unit of weight is the same as the unit of force. So, the weight is measured in Newton.

The Earth's gravitational force is called the gravity. This force of gravity, according to the Newton's second law of motion, causes acceleration called the acceleration due to gravity 'g' in a freely falling body. Thus

$$\boldsymbol{F} = m_G \, \boldsymbol{a} = m_G \, \boldsymbol{g} = \boldsymbol{W} \qquad (5.9)$$

The magnitude of the weight is given by

$$W = mg \qquad (5.10)$$

We shall see in later section that g changes from place to place. Therefore, the weight of a body also depends on the location of the body. ***The mass is an intrinsic property of a body but weight is the extrinsic property.***

Example 5.3 : *A mass of 15 g is suspended with a vertical spring which is extended by 15 division on a scale placed by its side. Find (a) the weight of the body and (b) the spring constant of the spring. Each division on the scale is equal to 1 mm.*

Solution: (a) $W = mg = 15 \times 10^{-3}$ kg $\times$ 9.8 ms^{-2} = 0.147 N.

The same result may be expressed in gravitational units in which

$W = 15 \times 10^{-3}$ kg wt

1 kg wt = 9.8 N.

(b) The spring exert restoring force $F = -kx$ where k is the spring constant. In equilibrium

$|F| = |W|$

$\therefore\ kx = mg$ or $k = mg/x$

Thus $k = \dfrac{15\times10^{-3}\,\text{kg}\times9.8\,\text{ms}^{-2}}{0.015\text{m}} = 9.8\,\text{Nm}^{-1}$

5.4.4 Properties of Inertial Mass

The inertial mass has following distinct properties,

(i) These can be added algebraically i.e. $M = m_1 + m_2 + m_3 + \ldots\ldots\ldots$;

(ii) It remains unaffected by the presence of other body's masses;

(iii) It is conserved during physical or chemical combinations;

(iv) It is independent of the shape, size and the state of the matter;

(v) It depends on the total quantity of the matter in the body;

(vi) Its value changes at speeds approaching the speed of light, then

$m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$, where m_0 is the rest mass of the body and c is the speed of light.

Again, take a break and check you progress.

INTEXT QUESTIONS 5.2

1. Distinguish between inertial mass and gravitational mass. Do you use the same balance to measure the two masses.

    ....

2. If $v << c$, will it make any difference if mass is measured by applying horizontal force or allowed it to fall freely?

    ....

3. A body is suspended with a spring balance attached to the ceiling of an elevator. The balance shows a reading of 5 division when the elevator is stationary. During downward acceleration of the elevator the balance shows zero reading. Do you think that the inertial and gravitation mass of the body are equal, justify your answer.

    ....

4. Why the weight of a body does not remain constant?

    ....

5. A bowler imparts a velocity of 10 ms^{-1} in 0.5 s to a cricket ball of weight 3 N. Find the horizontal force acting on the ball. Take $g = 10\ \text{ms}^{-2}$.

    ....

5.5 ACCELERATION DUE TO GRAVITY AND ITS VARIATION

We know that the acceleration due to gravity is the acceleration of a body falling freely. Its value was determined first of all by Galileo and he found that it was same for all bodies. In the inertial frame of reference, the gravitational force on a mass is given by equation (5.3) which is,

$$\boldsymbol{F} = -\frac{GM_E\, m}{R_E^2}\hat{\boldsymbol{r}}$$

According to the Newtons second law of motion

$\boldsymbol{F} = m\boldsymbol{a}$

Comparing these two equations we find

$$\boldsymbol{a} = -\frac{GM_E}{R_E^2}\hat{\boldsymbol{r}} \qquad (5.11)$$

The negative sign indicates that $\boldsymbol{r}$ and $\boldsymbol{a}$ are measured in opposite direction. We see that $\boldsymbol{a}$ is independent of m hence it has same value for all masses. This is known as the acceleration due to gravity $\boldsymbol{g}$. With the precision available in the measurement of length and time ($\sim 10^{-9}$), the value of g at 45° latitude is

$g = 9.80600\ ms^{-2} = 9.81\ ms^{-2}$

We may now write

$$|\boldsymbol{g}| = \frac{GM_E}{R_E^2} = \frac{F}{m} \qquad (5.12)$$

Example 5.4: *Two balls weighing 4.9 N and 19.2 N respectively are dropped from the same height of 10 m. Neglecting air friction, find how long each ball will take to reach the ground. Take g = 10 ms^{-2}.*

Solution: The equation of motion of freely falling body is

$h = \frac{1}{2} gt^2$, $[x = v_0 t + \frac{1}{2} at^2,\ v_0 = 0]$

as the balls are dropped. This gives

$$t = \sqrt{\frac{2h}{g}}$$

t is independent of mass, so both balls will take the same time to reach the ground. Putting the values of h and g we have

$$t = \sqrt{\frac{2\times 10}{10s}} = 1.4s$$

5.5.1 Variation in the value of *g*

A look on equation (5.12) reveals that the value of g depends on M_E and R_E. The value of g is not constant. It depends on some factors which are being discussed below. The Earth is not a perfect sphere, it is buldging at the equator. It is found that the value of g changes by about 0.6 per cent on the Earth. It also changes with altitude, depth and lattitude. Would you like to buy more gold for the same money?

(A) Variation of *g* with altitude : Let us take a body of mass m at point P at height h from the Earth's surface. (Fig 5.5). Its distance from the centre of Earth $OP = (R_E + h)$. Let the value of acceleration due to gravity at P be g'. Therefore,

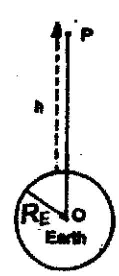


Fig 5.5: *Mass point P at an altitude h.*

$$mg' = \frac{G M_E m}{(R_E + h)^2} \quad or \quad g' = \frac{G M_E}{(R_E + h)^2}$$

On the Earth's surface

$$g = \frac{G M_E}{R_E^2}$$

Dividing these equations we have,

$$\frac{g}{g'} = \frac{(R_E + h)^2}{R_E^2} = \left(1 + \frac{h}{R_E}\right)^2 = 1 + \frac{2h}{R_E} + \left(\frac{h}{R_E}\right)^2$$

Since $R_E > h$, we can neglect $\left(\frac{h}{R_E}\right)^2$ so that

$$g = g'\left(1 + \frac{2h}{R_E}\right) \quad \text{or} \quad \boxed{g' = \frac{g}{\left(1 + \frac{2h}{R_E}\right)}} \qquad (5.13)$$

The above equation shows that $g' < g$. We find that with increase in altitude the value of g decreases, it is less than 9.8 ms^{-2}, the value on the surface of the Earth.

Example 5.5: *Calculate the percentage difference in the value of g at altitudes (a) 200 km and (b) 40 km. The radius of the Earth is 6400 km.*

Solution :

(a) $\frac{g'}{g} = \frac{R_E^2}{r^2} = \left(\frac{6400\,\text{km}}{6400 + 200\ \text{km}}\right)^2 = \left(\frac{6400}{6600}\right)^2 = 0.94$

$\Delta g = g - g' = 0.06$

So, the percentage variation = $\frac{\Delta g}{g} \times 100 = 0.61\%$

(b) $\frac{g'}{g} = \left(\frac{6400\ \text{km}}{6400 + 40\ \text{km}}\right)^2 = 0.9937$

$= \Delta g = 0.0063$

So, the percentage variation = 0.064%

Note that the value of g remains same near the surface of the Earth.

Example 5.6: *Taking radius of the Earth equal to 6.37×10^6 m and $g = 9.8$ ms^{-2} find the mass of the Earth. ($G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$).*

Solution: $M_E = \frac{g R_E^2}{G} = \frac{9.8\ \text{ms}^{-2} \times (6.37 \times 10^6\,\text{m})^2}{6.67 \times 10^{-11}} = 5.97 \times 10^{24}$ kg

$M_E = 5.97 \times 10^{24}$ kg

Density of the Earth

$$\rho = \frac{M_E}{V} = \frac{g R_E^2 / G}{\frac{4}{3}\pi R_E^3} = \frac{3}{4}\frac{g}{\pi G R_E}$$

$$= \frac{3\times 9.8\text{ms}^{-2}}{4\times 3.14\times 6.67\times 10^{-11}\text{ Nm}^{2}\text{kg}^{2}\times 6.37\times 10^{6}\text{ km}}$$

(B) Variation of g with latitude: Two factors, (a) position of the body above the surface of the Earth and (b) the latitude at which the body is situated affect the value of g. The first part has already been discussed. It is the rotation of Earth about its wn axis which gives rise to the second type of variation.

Consider a mass point P on the Earth's surface at a latitude λ as shown in Fig 5.6. Due to the rotation of the Earth from West to East on its own axis running North-South, the mass at P experiences a centrifugal force. It describes a circle of radius r, the radius of the Earth being R_E. Had the Earth been stationary $\boldsymbol{F}_G$ would act along PO. The mass point P describe a circle of radius

$r = R_E \cos\lambda$

Where λ is the latitude of P. The centrifugal force on P is

$|\boldsymbol{F}_C| = mr\omega^2$

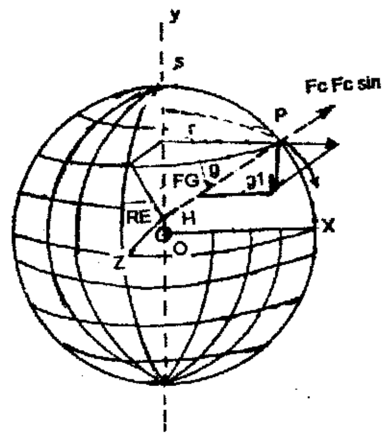


Fig. 5.6: *The effect of rotation of the Earth on the value of 'g'.*

From lession 1, you know that force can be resolved into two rectangular components namely $\boldsymbol{F}_C \cos\lambda$ along OP and $\boldsymbol{F}_C \sin\lambda$ tangential to the Earth at P. The tangential component has no effect along PO. The net force along PO is thus

$\boldsymbol{F}' = \boldsymbol{F}_G - \boldsymbol{F}_C \cos\lambda$

In lesson 4, you have learnt $F_C = mr\omega^2$, therefore, on writing magnitudes

$mg' = mg - mr\omega^2 \cos\lambda$

or $g' = g - R_E\omega^2 \cos^2\lambda$ (5.14)

We see that,

(i) the rotation of Earth decreases the value of g

(ii) the increase in the value of λ increases the value of g.

The value of g thus changes from place to place on the Earth, ***its maximum value is at the pole (λ = 90°) and minimum at the equator (λ= 0).***

Example 5.7: *Calculate the value of g at (a) the equator, (b) the poles, and (c) at a latitude of 45°. Given*

$(R_E)_{eq}$ = 6378.4 km,

$(R_E)_{pole}$ = 6356.9 km, M_E = 5.97 × 10^{24} kg.

Solution: At latitude λ, the value of acceleration due to gravity is

$g' = g - R_E u^2 \cos^2\lambda$

(a) At the equator λ = 0, cos 0 = 1, $\omega = 2\pi/T$

Therefore,

$$g_{eq} = \frac{GM_E}{(R_E)_{eq}^2} - (R_E)_{eq}\frac{4\pi^2}{T^2} \quad \left[\because g' = g - R_E\,\omega^2\right]$$

or $$g_{eq} = \frac{6.67\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}\times5.97\times10^{24}\,\text{kg}}{(6.3784\times10^6\,\text{m})^2} - \frac{6.3784\times10^6\,\text{m}}{(86400)\text{s}}$$

$= 9.78038\ \text{ms}^{-2}$

(b) At the poles $\lambda = 90°$, $\cos\lambda = 0$, $g' = g = GM_E/(R_E)^2_{pole}$

$$\therefore g_{pole} = \frac{GM_E}{(R_E)^2_{pole}} = \frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times5.97\times10^{24}\,\text{kg}}{(6.3569\times10^6)}$$

$= 9.854\ \text{ms}^{-2}$

(c) At latitude 45°, $\cos 45° = 1/\sqrt{2}$

$$g' = \frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times5.97\times10^{24}\,\text{kg}}{(6.371\times10^6\,\text{m})^2} - \frac{4\pi^2\times6.371\times10^6\,\text{m}}{(86400\ \text{s})}$$

$= 9.80620\ \text{ms}^{-2}$.

C) Variation of *g* with Depth : If we measure the value of *g* inside a mine, will it be the same as on the surface of the Earth? Let us explore it. Consider a point P at a depth h below the surface of the Earth as shown in Fig. 5.7. The mass at P may be considered to be situated inside a spherical shell of thickness h and on the surface of the sphere of radius $(R_E - h)$, where R_E is the radius of the Earth. The gravitational effect at P due to the shell is zero as the point P lies inside the shell. Therefore, the effective gravitational field at P is only due to the sphere of radius $(R_E - h)$. The earth is divided into a shell of thickness h and a symmetrical sphere of radius $(R_E - h)$.

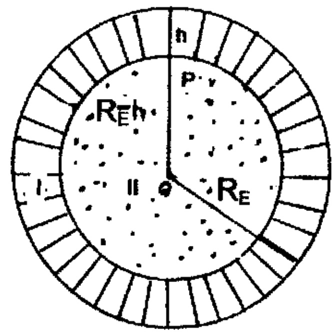


Fig 5.7: *A mass point P at depth h.*

The mass of the sphere = $\frac{4}{3}\pi(R_E - h)^3\rho$ If the value of acceleration due to gravity at P is g', then

$$g' = \frac{GM_E}{(R_E - h)^2} = \frac{G\frac{4}{3}\pi(R_E - h)^3\rho}{(R_E - h)^2} = \frac{4\pi}{3}G(R_E - h)\rho$$

Also $$g = \frac{GM}{R_E^2} = \frac{G\frac{4\pi}{3}R_E^3\rho}{R_E^2} = \frac{4\pi}{3}GR_E\rho$$

Dividing these two equations we have

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G(R_E - h)\rho}{\frac{4}{3}\pi GR_E\rho} = \frac{R_E - h}{R_E} = 1 - \frac{h}{R_E}$$

or $$\boxed{g' = g\left(1 - \frac{h}{R_E}\right)} \qquad (5.15)$$

We find that acceleration due to gravity decreases with increasing depth ($g' < g$). But it is important to note that variation of g with depth is not a simple affair .

At the centre of Earth $h = R_E$, then

$$g' = g\left(1 - \frac{R_E}{R_E}\right) = 0$$

The uneven variation in the value of g with depth is complex in nature and it depends on the density of the Earth at different depths.

Now, it is time to check your progress.

INTEXT QUESTIONS 5.3

1. The gravitational force on an object depends linearly on its mass, why is then the acceleration of a freely falling object independent of mass?

   ....

2. You are free to buy gold from a dealer at the equator or poles. To get maximum gold for your money would you like it to be weighed by a spring scale at the pole or equator?

   ....

3. In the British system the unit of length, mass and time are foot, slug and second respectively. What will be the unit of G in this sytem?

   ....

4. Determine the fractional decrease in the value of g due to increase in the elevation by 8 km near the surface of the Earth $R_E = 6400$ km.

   ....

5. The mass of asteroid Ceres is approximately 7×10^{20} kg and its diameter is 1100 km. What is the value of acceleration due to gravity at its surface? What would be the weight of an 80 kg astronaut on this asteroid?

   ....

5.6 KEPLER'S LAWS OF PLANETARY MOTION

The planet and stars in the sky have always attracted scientists to find out about these heavenly bodies. Throughout the last few decades of the sixteenth century, a Danish astronomer Tycho Brahe made precise measurements of the position of the planets and various others bodies of the solar system. Johannes Kepler made a detailed analysis of the measurements and announced three laws which describe planetary motion. These laws are thereafter called Kepler's laws of Planetary motion.

Law of orbits : *The orbit of any planet around the Sun is an ellipse with the Sun at one of the focii of the elipse.*

Law of areas : *The line joining the Sun and the planet, called the radius vector, sweeps equal areas in equal intervals of time.*

Law of periods : *For any two planets in the Solar system, the squares of the periods of revolution are proportional to the cube of their average distance from the Sun, i.e.*

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \quad \text{or } T^2 \propto r^3 \qquad (5.16)$$

Almost all planets have elliptical orbits and circle is a special case, (See Fig. 5.8). For most practical purposes we take the orbit nearly circular. In calculational work, in general, we consider planets in isolation as if the interaction of planets is not there (though the actual situation is different). We find that the speed of comets increases when they come closer to the Sun. So, is the case with the planets. They become slower when they move away from the Sun. It is explained by the law of areas.

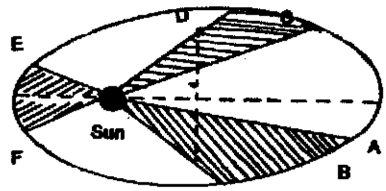


Fig 5.8: *Motion of planets around the sun in elliptical orbit.*
Area ASB = Area CSD = Area ESF

Example 5.8: *The mass of planet Jupiter is 1.90 × 10²⁷ kg and its radius is 7.14 × 10⁷ m. Calculate the acceleration of free fall on Jupiter. By what factor would your weight be larger than your weight on Earth?*

Solution:

$$g' = \frac{GM}{R^2} = \frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times1.90\times10^{27}\,\text{kg}}{\left(7.14\times10^{7}\,\text{m}\right)^2}$$

$= 22.9\ \text{ms}^{-2}$

Weight on Jupiter = $mg' = 22.9\ m\text{N}$
Weight on Earth = $mg = 9.8\ m\text{N}$

$$\text{Factor} = \frac{22.9\,\text{m}}{9.8\,\text{m}} = 2.33\ \text{times}$$

Example 5.9: *Calculate the mean distance from the Sun of hypothetical planets having periods (a) 50 year, (b) 100 year. Mean distance of the Earth from the Sun is equal to 149.6 × 10⁹ m.*

Solution: From Kepler's third law of motion we have

$$\frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2}$$

Consider the Earth as another body at a distance r^2 from the Sun. Now for the first hypothetical planet we write

(a) $r_1^3 = r_2^3\,(T_1/T_2)^2 = (149.6\times10^9\ \text{m})^3\,(50\ \text{y}/1\text{y})^2$
$= (149.6\times10^9\ \text{m})^3\ 2500$

So, $r_1 = 2.03\times10^{12}$ m

Here T_2 is taken as the period of the Earth about the Sun which is 1 year.

(b) For the second planet

$$\frac{r_3^3}{r_2^3} = \left(\frac{T_3}{T_2}\right)^3 \quad \text{or} \quad r_3^3 = r_2^3\left(\frac{T_3}{T_2}\right)^2$$

Thus, $r_3^3 = (149.6\times10^9\ \text{m})^3\ (100\ \text{y}/1\text{y})^2$
$= (149.6\times10^9)^3\ 10{,}000\ \text{m}^3$

or $r_3 = 3.2\times10^{12}$ m.

Now, take a break and solve the following questions.

INTEXT QUESTIONS 5.4

1. Show that Kepler's second law is the law of conservation of momentum.

....

2. Does the Moon obey Kepler's laws of motion?

....

3. Does a comet move faster at aphehelion or perihelion?

....

4. 4 Astronomical observations shows that Mercury moves fastest and Pluto slowest, why is it so?

....

5. The ratio of the radii of the two Earth satellites A and B is $r_A/r_B = 2$.. What are the ratio of their (a) periods (b) acceleration and (c) speeds.

....

5.7 MOTION OF SATELLITES

Every one of you have seen the Moon. You also know that it revolves around the Earth in circular orbit. Like Earth, Naptune has one moon, Uranus has four moons, Saturn ten and Jupiter has maximum number of moons 12. These moons revolve round the respective planet and are called the ***natural satellites***. Thus, we can say that the *satellite is a body which moves around a planet*. There are ***artificial satellites*** also. Ever since Russia launched its first man-made satellite *Sputnik-I* in October 1957, the sky has been flooded with the artificial satellites, we have our own satellites in the same crowed. Let us now find how satellite can move around the Earth at a certain height.

An object thrown horizontally from some height hits the ground like any projectile. The motion of projectile has been discussed in lesson 4. What happens when the objected is thrown hard enough? It will escape leaving the Earth and go around in elliptical orbit (see Fig. 5.10). The orbit will be a circle for a certain velocity of projection. The object travels in the curved path due to gravity and forward momentum.

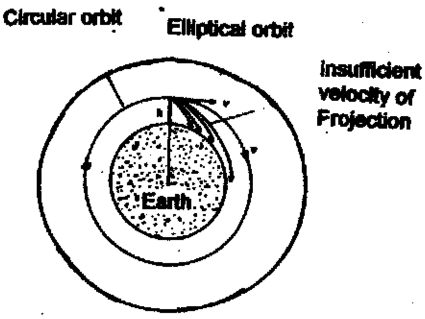


Fig 5.10 : *Projection of object with different speeds at different heights*

We see that a satellite may be put in an orbit provided (a) it is taken to a certain suitable height and (b) given a proper horizontal velocity so that it falls continuously without hitting the Earth.

The condition (b) is a characteristic of circular motion as shown in Fig 5.11. In the absence of centripetal force provided by the gravity, the object will fly off along a tangent in the direction of velocity. Such a situation can be visualised by whirling a small stone tied at one end of the string. If the hand stops pulling at the string the stone will fly off tangentially. Now decrease the length of the string by pulling a part of it in, we see that the stone now moves faster. Similarly satellites close to the Earth move faster.

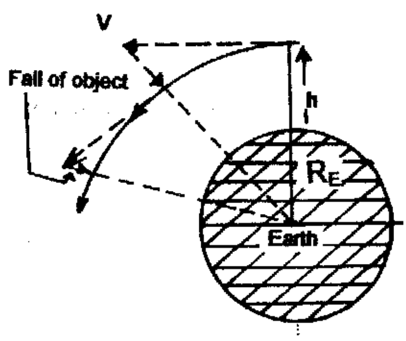


Fig 5.11: *The velocity is tangential at each point of the orbit and object continuously falls towards Earth to remain in circular orbit.*

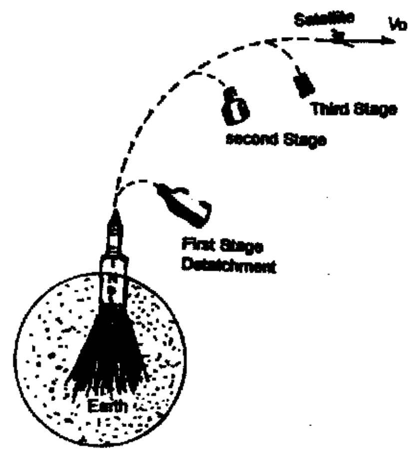


Fig 5.12 : *Satellite launch using a multistage rocket.*

5.7.1 (a) Orbital Velocity

The velocity with which a satellite or planet revolves in its orbit is called its orbital velocity. Like the orbit of a planet, for simplicity the orbit of the satellite is also taken as circular. As pointed out in the preceding section we need a velocity (v_0) to put the satellite in a desired orbit. Since gravity provides the centripetal force in the circular orbit of the satellite of radius r, we have

$$\frac{mv_0^2}{r} = G\frac{M_E m}{r^2}$$

$$\text{or} \quad v_0 = \sqrt{\frac{GM_E}{r}}$$

using eq. (5.12) we have $GM_E = gR_E^2$, so that

$$\boxed{v_0 = \sqrt{\frac{GM_E}{r}} = R_E\sqrt{\frac{g}{r}}} \qquad (5.17).$$

From eq. (5.17) we see that the orbital velocity is

(a) independent of the mass of the satellite

(b) inversely proportional to the square root of the radius of the orbit.

If the satellite is at a height h from the surface of the Earth then

$r = R_E + h$ and

$$v_0 = R_E\sqrt{\frac{g}{R_E + h}}$$

When the satellite moves close to the Earth $h = 0$ and $r \simeq R_E$, so

$$v_0 = \sqrt{gR_E} = 8\text{kms}^{-1} \quad (5.18)$$

Example 5.10: *Find the orbital velocity of the satellite around the Earth at heights (a) 70 km (b) 230 km. Take the radius of the Earth 6.37 × 10⁶ m.*

Solution:

(a) $r = R_E + h = (6370 + 70) \times 10^3 \text{ km} = 6.44 \times 10^6 \text{ m}$

$$v_0 = R_E\sqrt{\frac{g}{r}} = 6.37\times10^6 m\sqrt{\frac{9.8\text{ms}^{-2}}{6.44\times10^6\text{m}}} = 7.835\text{kms}^{-1}$$

Thus $v_0 = 8$ km s^{-1} which is the orbital velocity for a satellite revolving close to the Earth.

(b) $r = R_E + h = (6.37 + 0.23) \times 10^6 \text{ m} = 6.6 \times 10^6 \text{ m}$

$$v_0 = 6.37\times10^6\text{m}\sqrt{\frac{9.8\text{ms}^{-2}}{6.6\times10^6\text{m}}} = 7.385\text{kms}^{-1}$$

We find that even at an altitude of 230 km, the orbital velocity of the satellite is nearly same as for a satellite close to the Earth.

(b) Time Period of the Satellite

It is defined as ***the time in which the satellite makes one completes revolution around the earth.*** The time of revolution is, thus, equal to the circumfrance of the circular path divided by the orbital speed v_0

$T = 2\ \pi r/v_0$

Substituting for v_0 from eq. (5.17) we have

$$T = \frac{2\pi r}{\sqrt{\frac{GM_E}{r}}} = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

or

$$\boxed{T^2 = \frac{4\pi^2}{GM_E}r^3 = \frac{4\pi^2}{gR_E^2}r^3} \quad (5.19)$$

This is the basic equation of motion of a satellite.

When $r \simeq R_E$, i.e. for a satellite close to earth,

$$T^2 = \frac{4\pi^2}{g}R_E$$

5.7.2 Escape Velocity

You must have observed that when a body is thrown in the upward direction, it reaches a certain height and comes back to the earth. But when it is given greater initial velocity, it reacher greater height before coming back to the earth. If the body is given certain minimum initial velocity from the surface of the earth, so that it goes beyond the gravitational field of earth, then, the velocity is said to be the ***escape velocity***. In other words the ***escape velocity can be defined as the velocity which will***

take the projectile (body) to the infinite distance away above the surface of the earth when projected upwards.

Let us calculate its value. We know the work needed to take a mass m from the surface of the earth to infinity is equal to $\frac{GM_E\, m}{R_E}$, where M_E is the mass of the earth and R_E is the radius of the earth.

If a body is to be able to do this amount of work (and so to escape), it needs to have at least this amount of kinetic energy at the moment it is projected. Hence, the minimum velocity v_{es} to be given to a body so as to escape earth's pull is given by $\frac{1}{2} m\, v_{es}^2 = \frac{GM_E\, m}{R_E}$.

$$v_{es} = \left(\frac{2GM_E}{R_E}\right)^{1/2} = \sqrt{\frac{2GM_E}{R_E}}$$

Since $GM_E = gR_E^2$, we have

$$v_{es} = \sqrt{\frac{2gR_E^2}{R_E}}, \quad \text{or} \quad \boxed{v_{es} = \sqrt{2g\, R_E}} \qquad (5.20)$$

using equation (5.18) we have

$$\boxed{v_{es} = \sqrt{2}\, v_0} \qquad (5.21)$$

Taking v_0 = 7.91 km s^{-1}, we have

v_{es} = 11.2 km s^{-1}.

The escape velocity is thus about 3/2 times the orbital velocity.

Example 5.11: *A body is launched from the Earth (R_{es} = 1.5 × 10^{11} m). Calculate its escape velocity so that it gets out of the pull of the Sun.*

Solution:

$$v_{es} = \sqrt{\frac{2GM_s}{R_{es}}}$$

$$= \left[\frac{2 \times 6.67 \times 10^{-11}\,\mathrm{Nm^2\,kg^{-2}} \times 2 \times 10^{30}\,\mathrm{kg}}{1.5 \times 10^{11}\,\mathrm{m}}\right]^{\frac{1}{2}} = 4.22 \times 10^4\,\mathrm{ms^{-1}}$$

or v_{es} = 42.2 km s^{-1}

Compare it with the escape velocity for Earth (v_{es} = 11.2 km s^{-1}). Thus, it is much more difficult for a body to escape out of the solar system.

5.7.3 Geostationary Satellite

A satellite which remains fixed directly over a point on the surface of the Earth while revolving in its orbit is know as Geostationary or synchronous satellite. To an observer in the satellite, the Earth will appear stationary hence the name geostationary. In other words the time period of the satellite is exactly equal to time of rotation of the Earth on its axis, that is, one day (86400 s).

Ve know from eq. (5.19) that the period of revolution and the distance of he satellite from the Earth are related. Ve can find the height over the surace of the Earth of a satellite whose ime period is 1 day that is it becomes ι synchronous satellite. Using :q.(5.19) we may write,

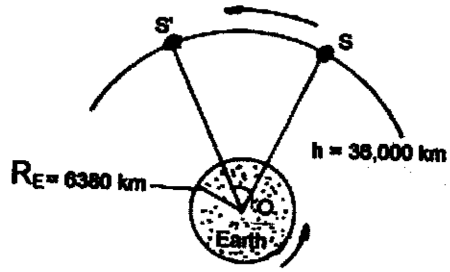


Fig 5.13 : *Two positions S and S' of a geostationary satellite at height 36,000 km over the surface of the Earth.*

$$r = \left[\frac{T^2}{4\pi^2} g R_E^2\right]^{\frac{1}{3}}$$

;ince $r = R_E + h$, therefore,

$$h + R_E = \left[\frac{T^2}{4\pi^2} g R_E^2\right]^{\frac{1}{3}}$$

$$\text{or } h = \left[\frac{T^2}{4\pi^2} g R_E^2\right]^{\frac{1}{3}} - R_E \qquad (5.22)$$

'aking $T = 1$ day = 86400 s, $R_E = 6.38 \times 10^6$ m and $g = 9.81$ ms^{-2}

$h = 42250 - 6380 = 35870$ km

or $h \simeq 36,000$ km.

. multistage rocket places the satellite at 36,000 km height over the surface f the Earth and a booster engine helps in fixing it in the circular orbit. It ; worth noting that the geostationary satellite is to be installed in the quitorial plane, otherwise its orbit will not be geostationary. In other osition the orbital plane changes continuously. INSAT I-B is an Indian eostationary satellite.

he advent of geostationary satellites have brought a sea change in the eld of communications. Three synchronous satellites can cover the whole lobal communication if they are placed at 120° with each other at an ltitude of about 36,000 km.

;.7.4 Application of Satellites

he satellites are being used in various fields for diverse purposes. The pplications are based on two things (i) infrared photography and (ii) relay. olar panels are used to supply energy to the satellite systems. Some of ıeir applications are given below.

. Communication satellite, such as INSAT series of satellites of India, are used in TV, radio telegraphy and radio communication.

. In guiding and tracking of missiles during war.

. These are used in remote sensing including
  (a) detection of troops and vehicular movement.
  (b) oil and mineral exploration
  (c) detection of military hardwares.

In weather forcasting.

5. These are used to locate atmospheric and oceanic disturbances such as cyclones and low pressure regions.
6. These are being used to determine the exact shape, dimensions and atmosphere of other planets and their moons.
7. These are being used in the environmental studies, pollution and population determination and crop pattern on the Earth.
8. Satellites are helping in charting ecology, forest and jungle cover, desert and glacier movements.

Example 5.12: *It takes 7 hour and 39 min for the satellite phobos to complete one round about the planet Mars. The orbital radius of Phobos is 9.4 × 10⁶ m. Find the mass of Mars.*

Solution: The relation between orbital radius and time period is given as

$$r^3 = \frac{T^2}{4\pi^2}GM$$

Assuming Mars to be stationary and $G = 6.67 \times 10^{-11}$ Nm² kg⁻²,
We have $T = (7 \times 3600 + 39 \times 60)$s.

$$M = \frac{4\pi^2}{T^2G}r^3 = \frac{4(3.14)^2 \times (9.4 \times 10^6 \text{ m})^3}{(2.754 \times 10^4 \text{ s})^2 \times 6.67 \times 10^{-11} \text{Nm}^2 \text{kg}^{-2}}$$

$= 0.6606 \times 10^{24}$ kg.

Stop, and check your progress by solving the following questions.

INTEXT QUESTIONS 5.5

1. Determine the mass of the Earth from the following data

Radius of the orbit of Moon	= 3.84×10^8 m
Period of the Moon	= 27.3 days
Gravitational constant (G)	= 6.67×10^{-11} Nm² kg⁻²

....

2. Assuming Jupiter to be stationary, find its mass. Its satellite Suropa has a period of 3.55 days and its orbital radius is 6.71×10^8 m.

....

3. Due to some unforseen event the orbital plane of the lo satellite of Jupiter does not pass through the centre of Jupiter. Will the orbit of lo be stable?

....

4. How high a man can jump on moon where acceleration due to gravity is 1/6 times acceleration due to gravity on the Earth.

....

5. Icarus is an asteroid behaving like a small planet. It can have a close approach to the Earth. Its aphelion distance is 2.946×10^8 km and perihelion distance is 2.8×10^7 km. Calculate the period of revolution of Icarus. Take Earth's mean distance from the sun = 1.496×10^8 km and its period 365 days.

[Hint : $r_1 = \frac{1}{2}(28.0 + 294.6) \times 10^6 \text{ km}, r_2 = 149.6 \times 10^6 \text{ km}$, Use Kepler's law of motion.]

....

5.8 WHAT YOU HAVE LEARNT

- The point masses attract each other anywhere in the universe. This force of attraction is called gravitational force.
- According to Newton's law of gravitation, the force of attraction between two point masses at a distance r is given by $\mathbf{F} = \frac{-Gm_1 m_2}{r^3}\mathbf{r} = \frac{-Gm_1 m_2}{r^2}\hat{\mathbf{r}}$
- The value of G is 6.67×10^{-11} Nm^2 kg^{-2} and it has dimensions of $(M^{-1} L^3 T^{-2})$.
- The gravitational force is a central force, it acts along the line joining the two point masses.
- The force of gravitation is both way i.e. m_1 attracts m_2 and m_2 attracts m_1 such that $\mathbf{F}_{12} = -\mathbf{F}_{21}$.
- The mass which characterises opposition (resistance) to acceleration under a force given by Newton's law $\mathbf{F} = m\mathbf{a}$ is known as inertial mass m_i.
- The mass which characterises the gravitational force on the body is called the gravitational mass m_G.
- Under ordinary conditions the inertial mass of a body and its gravitational mass are equal. However, if the speed of motion becomes comparable to the speed of light the inertial mass will change.
- Weight is the gravitational force acting on a body due to the Earth (planet) $= mg$.
- The acceleration caused by gravity in a freely falling body is called acceleration due to gravity 'g'.
- The value of 'g' at altitude h is given by $g' = g(1 - 2h/R_E)$
- The value of g changes with latitude λ and is given by

  $g' = g - \omega^2 R_E \cos^2 \lambda$
- The motion of planets around the Sun is governed by Kepler's law.

  First Law : Each planet moves around the Sun in elliptical orbits.

  Second Law : The line joining the Sun and the planet sweeps equal areas in equal intervals of time.

  Third Law : The square of the time period of revolution is directly proportional to the cube of its average distance from the Sun. $T^2 \alpha\ r^3$

  These laws are equally applicable to the motion of satellites as well.
- The orbital velocity of a satellite is given by

  $$v_0 = \sqrt{\frac{GM_E}{r}} = R_E\sqrt{\frac{g}{r}}.$$

  For satellites close to the Earth orbital velocity is $v_0 \cong \sqrt{gR_E} \approx 8\ \text{km s}^{-1}$
- Escape velocity is the velocity of projection on the Earth with which a body can escape out of the gravitational field of the planet or the Sun.

  $$v_{es} = \sqrt{2gR_E} = \sqrt{2}v_0 = 11.2\ \text{km s}^{-1} \text{ for Earth.}$$
- The period of motion of satellite is given by

  $$T^2 = \frac{4\pi^2}{GM_E}r^3 = \frac{4\pi^2}{gR_E^2}r^3$$
- Geostationary satellite is a satellite which remains fixed over a certain point on the surface of the Earth. Its period of rotation is equal to the period of rotation of the Earth about its own axis that is 1 day.
- The height at which a geostationary satellite is placed above the surface of the Earth is about 36,000 km.
- For a stable orbit, the geostationary satellite should remain in the equitorial plane.

5.9 TERMINAL QUESTIONS

1. State and explain Newton's law of gravitation.
2. Can gravitational force be repulsive?
3. Two artificial satellite, one loner to surface and the other away, are revolving round the earth, which one has longer period of revolution?
4. Which force keeps an artificial satellite revolving in its orbits?
5. Does the orbital velocity of a satellite depend upon its mass?
6. Is the escape velocity for two bodies of different masses on the surface of earth same or different?
7. State Kepler's laws of planetary motion.
8. Derive Kepler's laws using Newton's laws of gravitation and his laws of motion.
9. If two masses 50×10^7 kg and 40×10^7 kg respectively are placed at a distance of 6.67 m, calculate the force of attraction between.
10. Assuming Earth to be spherically symmetrical, determine the value of g at a height 0.13×10^6 m above its surface. Take the mass of the earth $M_e = 5.97 \times 10^{24}$ kg and its radius $R_E = 6.37 \times 10^6$ m.
11. Would you be able to stand on Jupiter where the value of acceleration due to gravity is about 3 times that on the Earth?
12. There is a planet called Egabbac in another solar system whose radius is twice that of the Earth but mass density is same as that of the Earth. If the acceleration of a falling body on that planet is 19.6 ms^{-2}, does it obey the law of gravitation?
13. Show that the mass of a planet is given by $m_p = r_p^2\, g_p/G$, where g_p is the acceleration due to gravity on the surface of the planet of radius r_p.
14. Find the period of a satellite 40,000 km from the centre of Earth. Mass of Earth = 5.98×10^{24} kg.
15. Considering the mass of the Moon as 7.4×10^{22} kg and radius as 1740×10^3 m, calculate the value of g on the surface of the moon.
16. The acceleration of gravity at the surface of a planet is half that on the surface of the earth. If the radius of the planet is half the radius of the earth, how is its mass related to the mass of the earth?
17. An astronaut weighs 100 kg on the earth. What is his weight on a planet x which has a radius $R_x = \dfrac{R_E}{2}$ and a mass $M_x = \dfrac{M_E}{8}$?
18. Calculate the escape velocity on the surface of the moon. Given, mass of the moon = 7.35×10^{22} kg and radius of moon = 1.60×10^6 m.
19. An artificial satellite is revolving around the earth at height of 800 km above the surface of the earth. Find its orbital velocity and the time period of revolution.
20. Mars has a mean diameter of 6,720 km and earth of 12,800 km. The mass of Mars is 0.11 times the mass of earth.
    (a) What is the escape velocity on Mars?
    (b) How does the mean density of Mars compare with that of earth?
    (c) What is the value of g on Mars?

5.10 ANSWER TO THE INTEXT QUESTIONS

INTEXT QUESTIONS 5.1

1. All bodies are considered to be point masses
2. The acceleration of m_2 will be negligible ($m_1 a_1 = m_2 a_2$)
3. (i) $9F_G$ (ii) $F_G/16$
4. 6.64×10^{-11} Nm^2 kg^{-2}
   [Hint : In equilibrium condition the deflection couple = torsional couple
   Torque = force × moment arm, i.e. $\tau = F_G\, l$
   also Torque $\tau = C\theta$, Equating we have $F_G\, l = C\theta$]
5. 3.41×10^8m

INTEXT QUESTIONS 5.2

1. The two masses are measured by different techniques.

2. No difference.
3. The two masses will be equal as the speed of the elevator is small compared to the speed of light.
4. Weight depends on the value of g which varies with place, altitude depth etc.
5. 6N

INTEXT QUESTIONS 5.3

1. $a = F/m$ is constant as $F \propto m$
2. At the equator, as the weight will be measured less there.
3. lb-ft^2 (slug)$^{-2}$
4. 2.5×10^{-3}
5. 0.15 ms^{-2}, 12 N

INTEXT QUESTIONS 5.4

1. Areas of Triangle made by radius $\frac{1}{2} r_1 v_1 = \frac{1}{2} r_2 v_2$
   So, $mr_1 v_1 = mr_2 v_2$

(b) Yes ratio r^3/T^2 is almost constant.

(c) Venus and Naptune

(d) Mercury and Pluto

(e) 1.33×10^{20}, 1.32×10^{20}, 1.327×10^{20}, 1.33×10^{20}, 1.33×10^{20}, 1.32×10^{20}, 1.34×10^{20}, 1.32×10^{20}, 1.33×10^{20} m^3 s^{-2}

The result shows that $a_c R^2$ is constant which proves Keplers third law of motion and also Newton's law of gravitation is proved.

2. Yes
3. At the perihelion that is when it is closest to the Sun.
4. Because planets obey Kepler's third law of motion.
   $T^2 \propto r^3$, mercury has minimum r and Pluto maximum $v = 2\pi r/T$.
5. (a) $2\sqrt{2}$, (b) ¼, (c) $1/\sqrt{2}$

INTEXT QUESTIONS 5.5

1. 6.04×10^{24} kg.
2. 1.90×10^{27} kg.
3. No.
4. 6 times higher compared to the Earth.
5. 409 days.

EXTENDED LEARNING

Gravitational Field

Gravitational field is defined as some modification or a condition in the space around a point mass. This field acts on any other mass m placed anywhere in the field. The field intensity is defined as the force acting on a unit mass in the field. So, the field intensity I is $I = F/m$. On the Earth $I = \frac{F_G}{m} = g$

The gravitational field intensity or simply field is thus, $g = -\frac{GM}{R^2}\hat{r}$

It is a vector field which acts towards mass M. The general form of the field intensity is given by $g = -\frac{GM}{r^3} r$

It has the dimensions of acceleration and is independent of the mass point m but depends on the mass M which has created the field. The force acting on a point mass m in the field is $F = m\,g$

Gravitational Potential Energy

The gravitational potential energy of a point mass is due to its location in the gravitational field. The potential energy difference is found by calculating the work done in taking a unit point mass from one point in the field to another point against the gravitational field. It can be shown either by numerical method or integration that the potential energy of a mass m in the field of mass M at a distance r from it is given by $U = -G\frac{Mm}{r}$

It must be noted that the P.E is taken zero at infinity.

Gravitational Potential

We know that the earth attracts everybody towards its centre. If a body is to be taken from any point near its surface to a far off point, work has to be done on it against earth's gravitational pull. However, if the reverse process is to be accomplished i.e. the same body is to be brought from a far off point to a point near the surface of the earth, work is done on it by the earth. The total amount of work done on a unit mass body to bring it from infinity to a point near the surface of earth is called gravitational potential at that point.