8

PROPERTIES OF SOLIDS

8.1 INTRODUCTION

In the previous lessons we have studied about force and energy. The effect of force applied to a body is observed either by displacement produced in the body or by change in its shape or size or both. It is our common experience that when a force is applied on a rubber cord or metal spring along its length, the change in length takes place. When we remove the force, the cord regains its original length. When a weight is put on thick foam sheet it is depressed but after removing the weight it regains its original shape. Further you can easily see that in case of a ball of wet model clay or molten wax it changes its shape or size or both even on application of a very small force and does not regain its original condition after removing the force. Do you know why some bodies regain their origin shape and size whereas others do not?

In this lesson we will be studying about the elasti propoerty of the material.

This property of the materials is of vital importance in our lives. In this lesson we will also study that elastic property of material is for calculating the strength of cables (or strings) used to suspend a body such as a cable car, crane, lift, etc. We will also learn its uses for finding the strength of beams for construction of buildings and bridges.

8.2 OBJECTIVES

After studying this lesson, you should be able to :

- *explain molecular theory of matter and distinguish between three states of matter on its basis;*
- *distinguish between elastic and plastic bodies;*
- *define the terms deforming force, stress, strain and their types;*
- *distinguish between stress and pressure;*
- *draw and interprete stress-strain curve for an elastic solid;*
- *define Young's modulus, bulk modulus, modulus of rigidity and Poisson's ratio;*
- *explain characteristics of a cantilever;*
- *state the dependence of the depression of a loaded cantilever and girder on various parameters; and*
- *explain the peculiar shape of girder having cross-section in the form of letter I.*

8.3 MOLECULAR THEORY OF MATTER

You are familiar that every matter is made up of molecules. The molecules attract each other with a certain force. This force is called **intermolecular force.** As the separation between molecules decreases, the net attraction force between them increases. But when the molecules come very close to each other, the attraction force between them begins to decrease; and when the distance becomes too small, the molecules begin to repel each other. If molecules come still closer, the repulsive force increases more rapidly compared to attractive force.

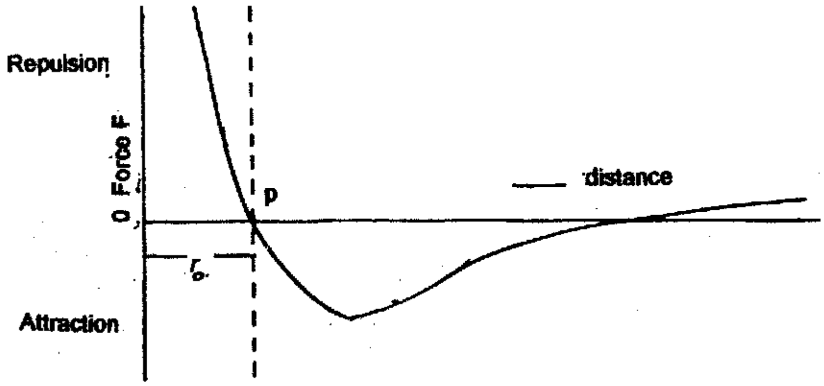

Fig. 8.1 : *Graph between intermolecular force and intermolecular separation*

The variation of intermolecular force with the variation of intermolecular distance is shown in Fig. 8.1. At separation r_0 net force between molecules is zero. This separation r_0 is called **equilibrium separation.** The magnitude of r_0 is of the order of 10^{-10} m. Thus, when the distance between two molecules is greater than r_0, then they attract each other but when the distance between them is less than r_0 they repel each other.

In **solids** molecules are close to each other very nearly at the equilibrium separation. Due to high intermolecular forces they are almost fixed at their positions. You may understand why a solid has a fixed shape if no external forces act to deform it.

In **liquids** the average separation between the molecules is somewhat larger. The attractive force is week and the molecules are more free to move inside the whole mass of the liquids. You may guess why liquids do not have fixed shape. It takes the shape of vessel in which it is filled.

In **gases**, the separation is much larger and the molecular force is very weak. Molecules of a gas are much more free to move inside as well as outside (if possible) the mass of the gas. Now can you think why gases do not have fixed shape and size?

8.4 ELASTICITY

As you know that a rubber cord or metalic spring is tied at one end and a force is applied at the other end, by hanging weight there is a change in

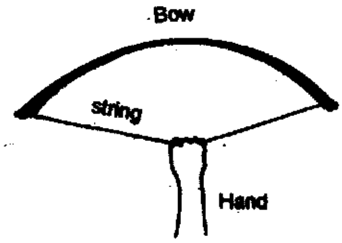

Fig. 8.2: *Force applied on string of bow*

its length and a removing the weight it comes in its original condition. On pressing a rubber ball, its shape is changed and on releasing it becomes in its original shape. Similarly when you apply a force on the string of a bow Fig 8.2, it is stretched and produce deformation in the bow. On releasing the bow it comes back to its original shape. From these examples we infer that :

(i) *Whenever an external force is applied on a body its shape or size or both change. In other words the body is deformed. The extent of deformation depends upon the material and shape of the body and the manner in which the force is applied.*

(ii) *on releasing external deforming force, the body tries to regain its original state.*

The property of matter to restore the natural shape and size or to oppose the deformation is called elasticity.

8.4.1 Elastic and Plastic Bodies

If you pull a spring, it is extended but on releasing it comes back exactly to its original state (length, etc.). The body which recovers completely its original state on the removal of the deforming force is called **perfectly elastic**. On the other hand if it completely retains its modified form even on removing the deforming force, i.e., it shows no tendency to recover, it is said to be **perfectly plastic.** However, it is important to note that all bodies behave in between these two limits. Below a certain limit of deforming forces **quartz** may be considered perfectly elastic and above this limit **plasticin** is taken as perfectly plastic. Do not confuse the plastic used in daily life with the plastic body. No doubt elastic deformations are very important in science and technology but plastic deformations are also important in mechanical processes. You may see the processes such as stamping, bending and hammering metal work pieces. These are only possible due to presence of plastic deformations.

We are familiar with the structure of matter. The phenomenon of elasticity can be explained in terms of intermolecular forces.

8.4.2 Molecular Theory of Elasticity

As you are aware that a solid body is composed of a great many molecules or atoms arranged in a definite order. Each molecule is acted upon by the forces due to neighbouring molecules. Due to inter-molecular forces solid takes such a shape that each molecule remains in a stable equilibrium. When the body is deformed, the molecules are displaced from their original positions. The inter-molecular distances change. If in deformation intermolecular separation increases from their equilibrium separation

($r > r_0$) attraction forces are developed. Whereas if intermoleculear separation decreases ($r < r_0$), repulsion forces are developed. These forces called as **restoring forces** drive molecules to their original positions so that body takes its original shape and size.

Now you study the manners in which forces are applied to deform the body.

8.4.3 Stress

When an external force or system of forces is applied to a body, it undergoes change in shape or size according to nature of forces. We have explained that in the process of deformation, internal restoring force is developed due to molecular displacements from their positions of equilibrium which opposes the deforming force. ***The internal restoring force acting per unit area of cross-section of deformed body is called stress.***

In equilibrium the restoring force is equal in magnitude and opposite in direction to the external deforming force. Hence stress is measured by the external force per unit area of cross-section when equilibrium is attained.

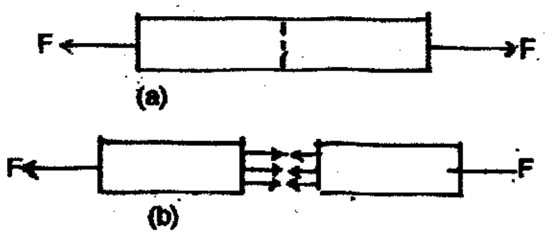

Fig. 8.3: *Longitudinal stress*

$$\text{Stress} = \frac{\text{restoring force}}{\text{area}} = \frac{\text{deforming force (F)}}{\text{area (A)}}$$

or $$\boxed{\text{Stress} = \frac{F}{A}} \quad ...(8.1)$$

The unit of stress is N/m^2. The stress may be longitudinal, normal or shearing.

(i) ***Longitudinal Stress*** : If the deforming forces are along the length of the body (fig. 8.3) then we call the stress produced as longitudinal stress.

(ii) ***Normal Stress*** : If the deforming forces are applied uniformly over entire surface of the body normally so that change in volume of the body occurs without change in shape (fig. 8.4). Then we call the stress produced as normal stress. You may produce normal stress by applying hydrostatic pressure uniformally over the entire surface of the body.

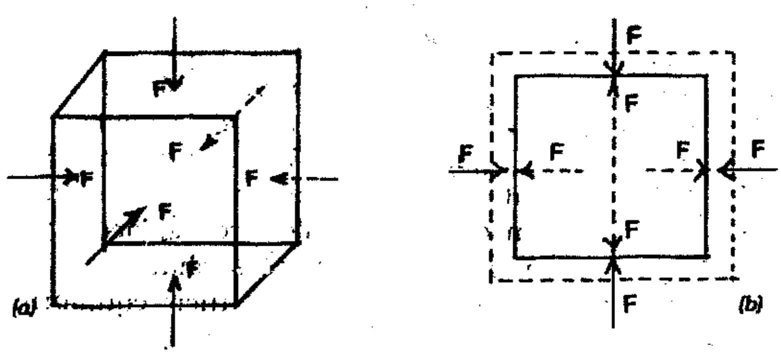

Fig. 8.4: *Normal stress (Change of volume without change of shape)*

Here you can distinguish pressure and stress. Deforming force acting per unit area normal to the surface is called pressure while internal restoring force developed per unit area normal to the surface is known as stress.

(iii) ***Shearing Stress*** : If the deforming forces act tangentially or parallel to the surface (fig. 8.5) so that shape of the body changes without change in volume, the stress is called shearing stress.

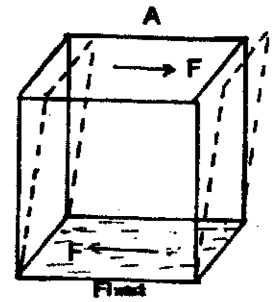

Fig.8.5: *Shearing Stress change in shape without change of volume)*

8.4.4 Strain

Deforming forces produce changes in the dimensions of the body. Corresponding to three types of stresses, these changes are also of three types.

In general the ***strain is defined as the change in dimension per unit dimension of the body.*** It is the ratio of two similar quantities hence is dimensionless.

As indicated above strains are of three types : (i) linear strain, (ii)volume (bulk) strain, and (iii) shearing strain.

(i) ***Linear strain*** : If on application of longitudinal deforming force length l of the body changes by Δl (Fig. 8.6) then

$$\text{linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l}$$

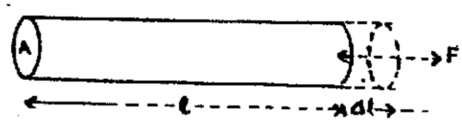

Fig.8.6: *Linear strain*

(ii) ***Volume strain*** : If on application of uniform pressure, the volume V of the body changes by ΔV (fig. 8.7) without change of shape of the body, then

$$\text{volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

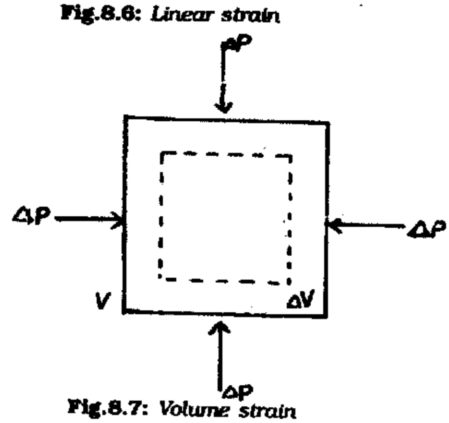

Fig.8.7: *Volume strain*

(iii) ***Shearing strain*** : When the deforming forces are tangential (fig. 8.8) then ***the shearing strain is given by the angle θ through which a line perpendicular to the fixed plane is turned after deformation.***

For small angle, $\theta = \dfrac{\Delta x}{y}$

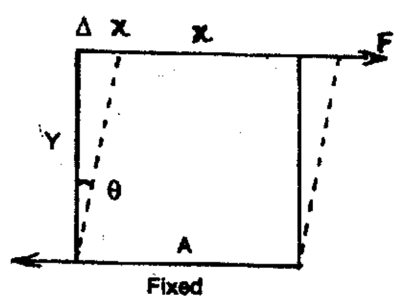

Fig.8.8: *Shearing strain*

8.4.5 Stress-strain curve for a metallic wire

Fig. 8.9 shows a graph showing variation of stress with the variation of strain when a metallic wire of uniform cross-section is subjected to an increased load.

Several regions and points on this curve are of great importance.

(i) ***Region of Proportionality (OA) :*** Portion OA of curve is a straight line which indicates that in this region stress is proportional to strain and body behaves like a perfectly elastic body.

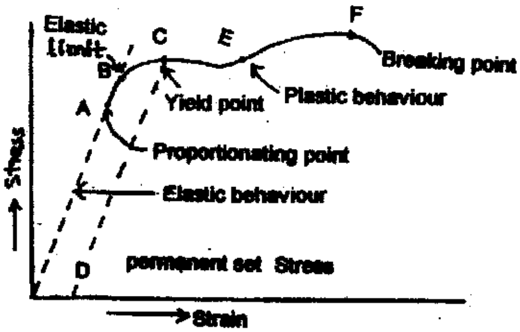

Fig.8.9: Stress-strain curve for steel wire

(ii) ***Elastic Limit (B) :*** If we increase the strain a little bit beyond A, the stress is not exactly proportional to strain. However, the wire still remains elastic i.e., after removing the deforming force (load), it regains its original state. ***The maximum value of strain for which a body (wire) shows elastic property is called elastic limit.*** Body beyond the elastic limit behaves like a plastic body.

(iii) ***Yield point (C) :*** When the wire is stretched beyond the limit B, strain increases more rapidly and body becomes plastic, that is, if the deforming load is removed, wire will contract but all the extension is not recoverable. The material follows dotted line CD on the graph during contraction and the remaining strain OD is known as a **permanent set**.

After point E on the curve none of the extension is recoverable.

(iv) ***Breaking point (F) :*** Beyond point E strain increases much more rapidly and near point F the length of wire increases continuously even without increase of force or even reducing the force a little. The wire breaks at point F . This is called as breaking point or fracture point.

The stress corresponding to breaking point F is called ***breaking stress*** or *tensil strength.*

If large deformation takes place between the elastic limit and the breaking point, the material is called ***ductile.*** If it breaks soon after the elastic limit is crossed, it is called ***brittle.***

8.4.6. Stress-strain curve for rubber

If you stretch a rubber cord to over several times its original length, even then after removing stretching forces, it will come back to its original length. The stress-strain curve for rubber is distinctly different from that of a metallic wire. There are two important things to note from fig. 8.10. Firstly you can observe that no part of this large deformation stress is proportional to strain. There is no region of proportionality. Secondly when the deforming force is removed, the original curve is not retraced although the sample finally acquires its natural length. The work done by the material in returning to its original shape is less than the work done by the deforming force when it was deformed. Thus, a certain amount of energy is absorbed by the material in cycle which appears as heat. This phenomenon is called ***elastic hysteresis.***

Elastic hysteresis has an important application in shock absorbers. A part of energy transferred by deforming force is observed in a shock absorber, only a small part is transmitted to the body.

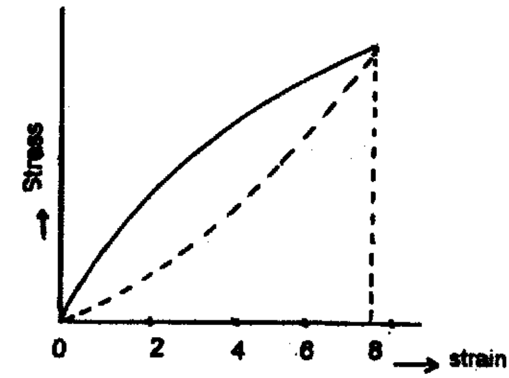

Fig.8.10: *Stress-strain curve for rubber*

Example 8.1 : *A load of 4.0 kg is suspended from a ceiling through a steel wire of radius 2.0 mm. Find the tensil stress developed in the wire when equilibrium is achieved.* Given $g = 3.1\ \pi m/s^2$

Solution : Tension in the wire

$$F = Mg = 4.0 \times 3.1\ \pi\ N$$

Area of cross-section of the wire

$$A = \pi r^2 = \pi\ (2.0 \times 10^{-3})^2\ m^2$$

$$= 4.0 \times 10^{-6}\ \pi\ m^2$$

The tensil stress developed $\dfrac{F}{A} = \dfrac{4.0 \times 3.1\pi}{4.0 \times 10^{-6}\pi}\dfrac{N}{m^2}$

$$= 3.1 \times 10^{-6} N/m^2$$

Example 8.2 : *For steel the breaking stress is $7.9 \times 10^6\ N/m^2$ and density is $7.9 \times 10^3\ kg/m^3$. Find the maximum length of a steel wire which can be suspended without breaking under its own weight ($g = 9.8\ m/s^2$).*

Solution : Let L be the maximum length of wire suspended without breaking. If ρ be the density and A the area of cross-section of the wire, then weight of the wire is

$$W = AL\rho g$$

Due to this weight the stress developed is $\dfrac{W}{A} = L\rho g$

This is the breaking stres. Thus

$$L\rho g = 7.9 \times 10^6\ N/m^2$$

or $$L = \frac{7.9 \times 10^6}{7.9 \times 10^3 \times 9.8} = 10^2\ m$$

Now it is the time to take a break and check your progress. For this try to solve the following questions.

INTEXT QUESTIONS 8.1

1. *If a rod is clamped rigidly at one end, a force is applied normally to the cross-section at the other end so that there is an increase in length of the rod, what is the type of strain in this case?*

   ....

2. *When a rope is used to pull a car it will be under compression or tension*

   ....

3. *The ratio stress/strain remains constant for small deformation of a metal wire. When the deformation is made larger, will this ratio increase or decrease ?*

   ....

4. *On applying a deforming force the inter-atomic separation in a body becomes lesser than that of inter-atomic seperation in the normal state. What must be the nature of the inter-atomic forces in this case ?*

   ....

5. *What type of change occurs in the body when shearing stress is developed in it ?*

   ....

6. *A wire is elongated by 8 mm by applying a force of 5 kg.wt. If its radius is doubled (other things remaining the same) the increase in its length will be*

   (a) 8 mm (b) 2mm (c) 32 mm (d) 1 mm

   ....

8.5 HOOKE'S LAW

In 1678 Robert Hooke obtained the stress-strain curve experimentally for a number of solid substances and established a law of elasticity known as Hooke's law after his name. According to this law : ***Under proportionality limit the stress is proportional to the corresponding strain.***

i.e., stress ∝ strain

or $\boxed{\text{stress/strain} = \text{constant } (E)}$ ...(8.2)

This constant E of proportionality is a measure of elasticity of the substance and is called **modulus of elasticity**. As strain is dimensionless quantity, modulus of elasticity has the same dimensions (or units) as that of the stress. Its value is independent of the stress and strain but depends on the nature of the material.

8.5.1 Modulii of Elasticity

In previous sections you have learnt that there are three kinds of strain.

It is therefore clear that there should be three modulii of elasticity corresponding to these strains. These are ***Young's modulus***, ***bulk modulus*** and ***modulus of rigidity*** corresponding to linear strain, volume strain and shearing strain respectively. Let us study them one by one.

(i) Young's Modulus: *The ratio of the longitudinal stress to the longitudinal strain is called Young's modulus for the material of the body.*

Let a wire of length L and area of cross-section A be stretched by a longitudinal force F. As a result change in length of the wire is Δl. Then

Longitudinal stress = F/A

Longitudinal strain = $\Delta l/l$

Young's modulus $Y = \dfrac{F/A}{\Delta l/l} = \dfrac{Fl}{A\Delta l}$

If the wire of radius r is suspended vertically with a rigid support and a mass M

hangs at its lower end, then $A = \pi r^2$ and $F = Mg$

$$\therefore \quad \boxed{Y = \frac{Mgl}{\pi r^2 \Delta l}} \qquad ...(8.3)$$

(ii) Bulk Modulus: *The ratio of normal stress to the volume strain is called bulk modulus of the material of the body.*

If due to increase in pressure Δp volume V of the body decreases by ΔV without change in shape, then

normal stress = Δp

volume strain = $\Delta V/V$

$$\text{Bulk modulus } B = \frac{\Delta p}{\Delta V/V} = V\frac{\Delta p}{\Delta V} \qquad ...(8.4)$$

The reciprocal of bulk modulus of a substance is called its **compressibility**.

$$\text{Thus, compressibility, } \boxed{K = 1/B = \frac{1}{V}\frac{\Delta V}{\Delta p}} \qquad ...(8.5)$$

(iii) Modulus of Rigidity or Shear Modulus: *The ratio of the shearing stress to shearing strain is called modulus of rigidity of the material of the body.*

If a tangential force F acts on an area A and θ is the shearing strain, then,

$$\text{modulus of rigidity } \eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} \qquad ...(8.6)$$

Example 8.4 : *A load of 4.0 kg is suspended from a ceiling through a steel wire of lengh 20 m and radius 2.0 mm. It is found that the length of the wire increased by 0.031 mm as equilibrium is achieved. Find Young's modulus for steel. Take* $g = 3.1\,\pi\ m/s^2$

Solution : Longitudinal stress $=F/A=Mg/\pi r^2$

$$=\frac{4.0\times3.1\pi}{\pi\left(2\times10^{-3}\right)^2}\ N/m^2$$

$$=3.1\times10^6\ N/m^2$$

Longitudinal strain $\frac{\Delta l}{l}=\frac{0.031\times10^{-3}}{2.0}=0.0155\times10^{-3}$

Thus, young's modulus $Y=\frac{3.1\times10^6}{0.0155\times10^{-3}}N/m^2=2.0\times10^{11}\ N/m^2$

Example 8.5 : *A 4.0 meter long copper wire of cross-sectional area 1.2 cm² is stretched by a force of 4.8 × 10³ N. If Young's modulus of copper is Y = 1.2 × 10¹¹ N/m², calculate (i) stress (ii) strain and (iii) increase in length of the wire.*

Solution :

(i) Stress $=\frac{F}{A}=\frac{4.8\times10^3}{1.2\times10^{-4}}N/m^2=4.0\times10^7\ N/m^2$

(ii) Strain $=\frac{\text{Stress}}{Y}=\frac{4.0\times10^7}{1.2\times10^{11}}=3.3\times10^{-4}$

(iii) Strain $=\Delta l/l$ increase in length Δl = strain × l
$= 3.3\times10^{-4}\times4\ m = 1.32\times10^{-3}\ m$

Example 8.6: *When a solid rubber ball is taken from the surface to bottom of a lake the reduction in its volume is 0.0012%. The depth of the lake is 360 m, density of lake water is 10³ kg/m³ and acceleration due to gravity is 10 m/s². Caculate bulk modulus of rubber.*

Solution:

Increase of pressure on the ball

$$p = h\rho g = 360\times10^3\times10\ N/m^2 = 3.6\times10^6\ N/m^2$$

Volume strain $\frac{\Delta V}{V}=0.0012/100=1.2\times10^{-5}$

Bulk Modulus $B=\frac{pV}{\Delta V}=\frac{3.6\times10^6}{1.2\times10^{-5}}=3.0\times10^{11}\ N/m^2$

8.5.2 Steel is More Elastic than Rubber

A body is said to be more elastic if on applying a large deforming force on it, the strain produced in the body is small. If we take two identical rubber and steel wires and apply equal deforming forces on each of them, you will see that the extension produced in the steel wire is smaller than the extension produced in the rubber wire. Also for producing same strain in the two wires the stress is much higher in steel wire than in rubber wire.

In defining linear strain or Young's modulus we have considered so far only increase in length of the wire on applying a stretching force and have completely ignored the resulting decrease in the diameter (thickness) of the wire. It was first of all pointed out by Poisson.

8.5.3 Poisson's Ratio

You may see that when a wire is stretched along its length, it is elongeted along with a contraction in its diameter. Thus, the length of the wire increases in the direction of forces, whereas a contraction occurs in the perpendicular direction. This fact is not only true for wire but for all other bodies under strain. The strain perpendicular to the applied force is called **lateral strain**. Poisson pointed out that within elastic limit lateral strain is directly proportional to longitudinal strain i.e. ***the ratio of lateral strain to longitudinal strain is a constant for a material body and is known as Poisson's ratio.*** *It is denoted by a Greek letter σ (sigma)*

If β and α be the lateral and longitudinal strains respectively, then the Poisson's ratio.

$$\sigma = \frac{\beta}{\alpha}$$

If a wire (rod or tube) of length l and diameter d is elongated on applying a stretching force by an amount Δl and its diameter is decreased by amount Δd, then longitudinal strain

$$= \Delta l / l$$

lateral strain $= \Delta d / d$

$$\therefore \text{ Poisson's ratio } \sigma = \frac{\Delta d / d}{\Delta l / l} = \frac{l}{d}\frac{\Delta d}{\Delta l} \qquad ...(8.7)$$

Since, Poisson's ratio is a ratio of two strains, it is dimensionless quantity and is purely a number.

The value of Poisson's ratio depends only upon the nature of the material and for most of the substances it lies between 0.2 and 0.4. When a body under tension suffers no change in volume, i.e., the body is perfectly incompressible, the value of Poisson's ratio is greatest, i.e., 0.5 (or $\frac{1}{2}$).

Example 8.7 : *A 10 kg mass is attached to one end of a copper wire 3 m long and 10^{-3} m in diameter. Find the extension if Young's modulus of the wire is 12.5×10^{10} N/m². Claculate the lateral compression produced if Poisson's ratio is 0.25*

Solution : Here $L = 3$ m, $r = 0.5 \times 10^{-3}$m, $Y = 12.5 \times 10^{10}$ N/m², $F = 10 \times 9.8$ N, $\sigma = 0.25$

Extension produced in the wire

$$\Delta l = \frac{F.l}{\pi r^2 Y} = \frac{10 \times 9.8 \times 3}{3.14 \times \left(0.5 \times 10^{-3}\right)^2 \times 12.5 \times 10^{10}} \text{ m}$$

$$= 0.2993 \times 10^{-2} \text{ m}$$

Longitudinal strain $\alpha = \Delta l / l = \frac{0.2993 \times 10^{-2}}{3}$

Poisson's ratio $\sigma = \frac{\text{lateral strain}(\beta)}{\text{longitudinal strain}(\alpha)}$

Lateral strain $\beta = \sigma \times \alpha$

$$= 0.25 \times 0.2993 \times 10^{-2}$$

$$= 2.4978 \times 10^{-4}$$

But, lateral strain $\beta = \frac{\Delta d}{d} = 2.4978 \times 10^{4}$

$$\Delta d = 2.4979 \times 10^{-4} \text{d}$$

$$= 2.4987 \times 10^{-4} \times 10^{-3}$$

Lateral compression $= 2.49879 \times 10^{-7}$ m

Now take another break and try to solve the following questions.

INTEXT QUESTIONS 8.2

1. *Is the unit of Young's modulus same as that of longitudinal stress or different?* ....

2. *Water is more elastic than air, why?* ....

3. *The length of a wire is cut to half. What will be the effect on the increase in its length under a given load?* ....

4. *Two wires are made of the same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of increase in their lengths* ....

5. *If the extension of a string is equal to its original length, then the Young's mnodulus of the material of string will be*

   (a) $= 2 \times$ *stress; (b)* $=$ *stress; (c)* $= \frac{1}{2} \times$ *stress; (d)* $= 3 \times$ *stress.*

6. *On applying a pressure of* $5 \times 10^6 N/m^2$ *the volume of a fluid of initial volume* $1.5 \times 10^{-3} m^3$ *changes by* 3×10^{-7}*. Bulk modulus of fluid is*

   (a) $7.5 \times 10^3 N/m^2$*; (b)* $2.5 \times 10^{10} N/m^2$*; (c)* $1.0 \times 10^3 N/m^2$*;*
   (d) $1.5 \times 10^{13} N/m^2$*.*

8.6 CANTILEVER

Beams are important in architecture, industries and also in our daily life. Elastic properties of matter provide load bearing strength to the beams (girders). ***A uniform beam clamped at one end and loaded at the other end is clalled cantilever.***

You would have seen Laxman Jhula in Rishikesh, Haridwar and Howara Bridge at Calcutta. These are examples of cantilever.

You will observe that when a long beam (rod or bar) is clamped horizontally at one end to a rigid support and a load is placed at the other end, it gets depressed.

You may easily observe experimentally the variation of depression of cantilever with the variation of load and length of cantilever. For this a long beam of length l and of uniform rectangular cross-section having breadth b and depth (thickness) d is taken. One end of it is clamped at the edge of a big table and a pointer is fixed to the free end which moves on a vertical scale (fig. 8.13) when it is loaded. At the free end weights may be suspended with the help of thread and hanger.

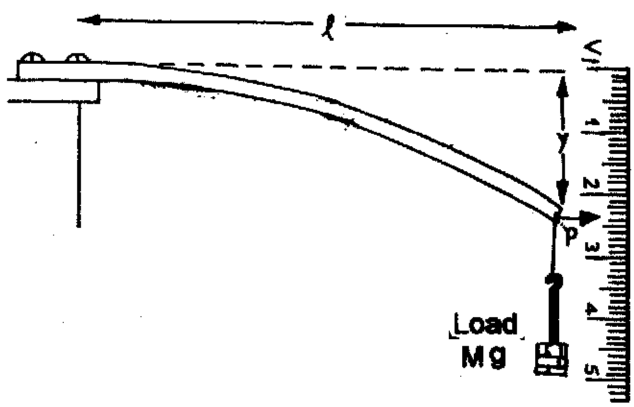

Fig. 8.13 : *Cantilever*

In order to study the variation of depression with load keeping length of contilever constant, position of pointer on the scale for zero load is noted. Then a weight of 50 g is suspended at free end. Due to weight, the beam bends and pointer moves downwards. Its position on scale is noted. The difference between two readings gives the depression. Then load is increased in steps of 50 g and several sets of observations are taken. With the help of these observations depression-load graph is plotted (fig. 8.14). A straight line graph shows that the ***depression produced is directly proportional to load provided the length of the cantilever is kept constant.***

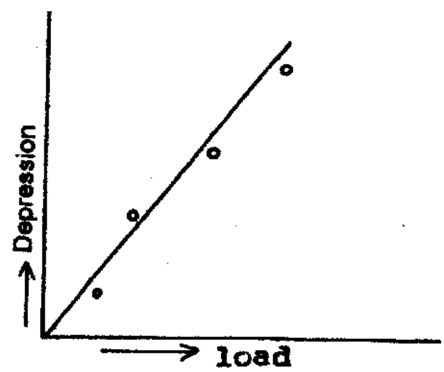

Fig. 8.14 : *Depression-load graph for a cantilever.*

In order to study the variation of depression with length of cantilever keeping the load constant, the length of the cantilever is changed in steps of 5 cm. and the corresponding depressions are noted keeping the load constant. In this case when the length of cantilever is altered the initial reading for the zero load is taken each time separately. Graphy of depression-length becomes a straight line (fig. 8.13).

The variation, of depression of cantilever may also be studied by drawing depression—breadth and depression-depth graphs.

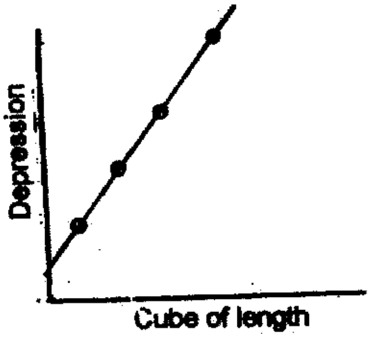

Fig. 8.13 : *Depression – Length graph for a cantilever*

Expression for Depression of Cantilever

For a beam of length *l*, breadth *b* and depth *d* the depression of the free end when it is loaded by weight of mass *M* is given by the expression.

$$\boxed{y = \frac{4Mgl^3}{Ybd^3}} \qquad \text{... (8.8)}$$

where Y is the Young's modulus for the material of beam.

> It is clear from the expression for depression of loaded end of a beam of rectangular cross-section that:
>
> (i) *for a given beam the depression is directly proportional to load (Mg.;*
>
> (ii) *for a given load the depression of the beam is*
>
> a) *directly proportional to the cube of its length.*
>
> b) *inversely proportional to its breadth.*
>
> c) *inversely proportional to the cube of its depth (thickness) and*
>
> d) *inversely proportional to Young's modulus of the material of beam.*

Thus, if you want small depression in a given cantilever it should not be loaded heavily. On the other hand if you want that for a given load the depression should be small, then the length of the cantilever should be small, its breadth, depth and Young's modulus (of the material) should be large.

8.6.1 Beam Supported at the Ends and Loaded in the Middle

Let us consider a beam of rectangular cross-section having length *l*, breadth *b* and depth *d* (fig.8.14). It is horizontally supported at knife edges k_1 and k_2 and load by wieght *W* at the middle point. Reaction of each of knife edges be *W*/2. It may be considered as double cantilever each of length $l/2$ and having load $W/2$. Thus, the depression of middle point below each of the knife edges is given by

$$y = \frac{4\left(\frac{W}{2}\right)\left(\frac{l}{2}\right)^3}{Y bd^3} = \frac{Wl^3}{4Y bd^3}$$

$$= \frac{Mgl^3}{4Y bd^3} \qquad ...(8.9)$$

Where M is the mass suspended at the middle

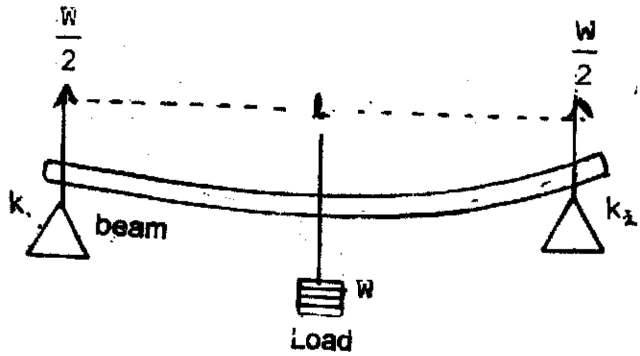

Fig. 8.14: Beam loaded in the middle as double cantilever

8.6.2 Explanation of Bending of Beams

Let us take a beam of rectangular cross-section. You may consider the beam as made up of a number of thin plane layers parallel to each other (fig. 8.15-a). Also each layer may be considered as made up of a number of parallel longitudinal filaments or fibres (fig. 8.15-b)

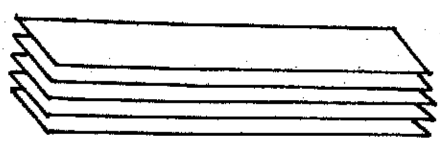
Fig. 8.15 (a) Layers in a beam

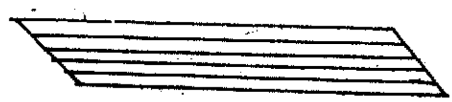
Fig. 8.15 (b) Filaments in a layer

Let the beam be clamped horizontally at end A and loaded by weight W at the other end B. It undergoes bending. The filaments of outward side of beam are elongated and become in tension while filaments of innerside are compressed (fig. 8.16). In between these two portions there is a layer or surface in which the filaments are neither elongated nor compressed. Such a surface is called ***neutral surface*** and is shown dotted in the figures.

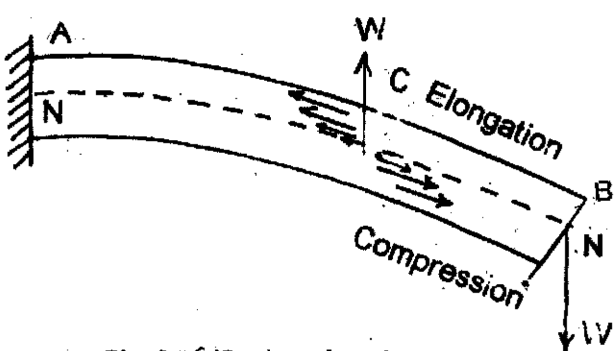

Fig. 8.16 Bending of cantilever

Consider the equilibrium of portion BC of the beam ; weight *W* at *B* and reaction force w at section C of beam form external couple which rotates the beam clockwise. Filaments above neutral surface increase in length and those below it decrease in length. The increase or decrease in length of a filament is directly proportional to its distance from the neutral surface. Due to elastic reaction internal forces are developed. These are shown with increasing magnitudes in fig. 8.16. A system of restoring couples is thus formed. The sum of moments of these couples about the neutral axis is called ***bending moment.*** In equilibrium bending moment is equal and opposite to the external couple.

8.6.3 Peculiar shape of Girders

Girders are used in the construction of buildings, bridges and factories. You might have noticed that the upper and lower faces of the girders are made of much greater breadth than its middle part which is of smaller breadth. Have you ever tried to know the reason of this?

When a girder is supported at its ends and is loaded, it is depressed in the middle due to bending. The filaments above neutral surface (middle layer is which there is neither contraction nor elongation of filaments on bending) are compressed and filaments below the neutral surface are elongated. Amount of compression of elongation increases as the distance from neutral surface increases.The compression and elongation are maximum for the upper most and lower most filaments respectively. Therefore outer layers suffer more strain and stress than inner layers. That is why, to make outer layers stronger than inner layers, the girders are manufactured with their cross-section in the form of letter I (fig. 8.17). This considerably saves the material without sacrificing the strength of the girder.

Fig. 8.17: *Shape of cross-section of girder*

Girders of Rectangular cross-Section: You must be knowing that in a girder of rectangular cross section the longer side is kept as depth.

The depression y of middle point of a girder of length l breadth b and depth d with a certain load W distributed uniformly along the length of the girder is given by

$$\boxed{y = \frac{5Wl^3}{32bd^3Y}} \qquad ...(8.10)$$

Where Y is the Young's modulus of material of the girder. It is clear that for the same load and length of the girder the depression y of middle point is inversely proportional to one power of breadth b and three powers of depth d. *Therefore, to reduce the depression increase in depth d is more effective than the increase in breadth b.* That is why in the girder of rectangular cross-section, the longer side is taken as depth.

Now let us check how much you have understood. Solve the following questions.

INTEXT QUESTIONS 8.3

1. *Are there certain filaments in a beam which remain unstrained during bending? Where do they lie?* ....
2. *How does the extension or contraction of filament change with different layer?* ....
3. *Two identical cantilevers are loaded by weights in the ratio 1:2. What is the ratio of depressions?* ....
4. *The length of a cantilever is halved. What will be the depression when load remaining the same?* ....
5. *A 5 m long beam of rectangular cross-section 10 cm × 15 cm. is to be used as a cantilever. Which side of the beams cross-section must be kept horizontal in order to have lesser depression?* ....

8.7 WHAT YOU HAVE LEARNT

- A force which causes deformation in a body is called deforming force.
- On deformation internal restoring forces are produced in the body so that after removal of deforming forces body regains its original shape and size.
- The property of matter to restore the natural shape and size after deformation or to oppose the deformation is called elasticity.
- The body which recovers completely its original state on the removal of the deforming forces is called perfectly elastic.
- If a body completely retains its modified form, it is said to be perfectly plastic.
- The stress equals the internal restoring force per unit area. Its units is N/m^2.
- The strain equals the change in dimension per unit dimension. Strain has no unit.
- In the normal state the net inter atomic force on an atom is zero. If the separation between the atoms becomes more than the separation in normal state. the inter atomic forces become attractive. However, for smaller separation these forces become repulsive.
- The maximum value of stress upto which a body shows elastic property is called its elastic limit. A body beyond the elastic limit behaves like a plastic body.
- Hooke's law states that within proportionality limit the stress developed in a body is proportional to the strain.
- The breaking stress is the stress for which a body breaks.
- Young's modulus is the ratio of longitudinal stress to longitudinal strain.
- Bulk modulus is the ratio of normal stress to volume strain.
- Modulus of rigidity is the ratio of the shearing stress to shearing strain.
- Poisson's ratio is the ratio of lateral strain to longitudinal strain.
- Steel is more elastic than rubber.
- A cantilever is the beam supported at one end and loaded at the other end.
- A beam supported at ends and loaded in the middle may be regarded as double cantilever.
- For a given cantilever depression produced is directly proportional to the load.
- For constant load depression of cantilever is directly proportional to cube of its length.
- For less depression in a cantilever of rectangular cross-section by a load the longer side of its cross-section is kept vertical.

- To save material without sacrificing the strength of girder they are manufactured with their cross-section in the form of letter I.

8.8 TERMINAL QUESTIONS

1. Define the term elasticity. When is a body called elastic and when is it called plastic?.
2. What is the unit of modulus of elasticity?
3. Explain the terms stress, strain, elastic limit and Hooke's law.
4. What is the value of young's modulus for a perfectly elastic body?
5. Explain the elastic properties of matter on the basis of inter atomic forces.
6. What is the value of bulk modulus for incompiressible liquid?
7. Draw a stress-strain graph for a metallic wire under increasing load. Mark the elastic limit on the graph. Which part of the graph is related to the Young's Modulus of the material of the wire?
8. Which is more elastic iron or rubber?
9. Define Young's modulus of elasticity.

   In order to produce a longitudinal strain of 2×10^{-4}, a stress of 2.4×10^7 N/m^2 is produced in a wire. Calculate the Young's modulus of the material of the wire.
10. Why poisson's ratio has no unit?
11. Define bulk modulus of elasticity.

    A solid cube of side 30 cm is subjected to a uniform pressure of 5×10 N/m^2. Calculate the change in its volume. Given that $B = 16.98 \times 10^{10}$ N/m^2.
12. What are shearing strain, shearing stress and modulus of rigidity? What is the unit of modulus of rigidity?
13. What are lateral strain, longitudinal strain and Poisson's ratio? A cylindrical wire of length l and radius r is stretched by a longitudinal force. Its length is increased by Δl and radius is decreased by Δr. Find out expression for Poisson's ratio for the material of wire.
14. A metallic wire 4 m in length and 1 mm in diameter is stretched by a weight of 4 kg. Determine the elongation produced. Given that the Y for the material of the wire is 13.78×10^{10} N/m^2.
15. A steel wire 2.8 m long and diameter 4×10^{-4}m stretched by a weight of 1.6 kg. If the extension produced is 1.66×10^{-3}m, calculate Y for steel.
16. The upperface of the cube of side 10cm is displaced 2mm parallel to itself when a tengential force of 5×10^5 N is applied on it keeping lowerface fixed. Find out strain.
17. What is a cantilever? If the free end of the beam is loaded, on what factors does the depression of the free end depend?
18. Girders are manufactured with their cross-section in the form of letter I. Give reason. What is the harm if the cross-section is made rectangular?
19. You have two bars of same length and same material but one of cross-section 6cm × 6cm and the other of cross-section 4 cm × 9 cm. Out of the two which one will you use as a cantilever? If you want to have minimum depression for a given load, what will you do?
20. A wire of length L and radius r is clamped rigidly at one end. When the other end of wire is pulled by a force F its length increases by x. Another wire of the same material of length $2l$ and radius $2r$, when pulled by a force $2F$, what will be the increase in its length.

ANSWERS TO THE INTEXT QUESTIONS

Intext Questions 8.1

1. Linear strain
2. Tension.
3. Decrease.
4. Repulsive.
5. Change of shape.
6. (b)

Intext Questions 8.2

1. Both have same unit.
2. Bulk modulus is reciprocal of compressibility. Air is more compressible than water. Therefore water is more elastic than air.
3. Half.
4. 1:8
5. (b)
6. (b)

Intext Questions 8.3

1. Filaments in the neutral layer remain unstrained during bending. They lie in the central layer of the cantilever.
2. Extension or contraction of filaments increased as their distance from the neutral surface increases.
3. For a loaded cantilever extension is maximum at upper most surface. Contraction is maximum at lower most surface.
4. 1:2
5. 1/8
6. 10 cm.

Terminal Questions

9. 1.2×10^{11} N/m^2
11. 7.95×10^{-5}m^3
14. 14.49×10^{-4}m
15. 2.105×10^{11} N/m^2
16. 2×0^{-2}
19. Bar of 4 cm × 9 cm cross-section, 9 cm side vertical.
20 Change in length is x