CARTESIAN SYSTEM OF COORDINATES

You must have searched for your seat in a cinema hall, a stadium, or a train. For example, seat $H-4$ means the fourth seat in the $H^{th}$ row. In other words, $H$ and 4 are the coordinates of your seat. Thus, the geometrical concept of location is represented by numbers and alphabets (an algebraic concept).

Also a road map gives us the location of various houses (again numbered in a particular sequence), roads and parks in a colony, thus representing algebraic concepts by geometrical figures like straight lines, circles and polygons.

The study of that branch of Mathematics which deals with the interrelationship between geometrical and algebraic concepts is called Coordinate Geometry or Cartesian Geometry in honour of the famous French mathematician Rene Descartes.

In this lesson we shall study the basics of coordinate geometry and relationship between concept of straight line in geometry and its algebraic representation.

OBJECTIVES

After studying this lesson, you will be able to:

- define Cartesian System of Coordinates including the origin, coordinate axes, quadrants, etc;
- derive distance formula and section formula;
- derive the formula for area of a triangle with given vertices;
- verify the collinearity of three given points;
- state the meaning of the terms : inclination and slope of a line;
find the formula for the slope of a line through two given points;
state the condition for parallelism and perpendicularity of lines with given slopes;
find the intercepts made by a line on coordinate axes;
define locus as the path of a point moving in a plane under certain conditions; and
find the equation of locus under given conditions.

**EXPECTED BACKGROUND KNOWLEDGE**

- Number system.
- Plotting of points in a coordinate plane.
- Drawing graphs of linear equations.
- Solving systems of linear equations.

### 9.1 RECTANGULAR COORDINATE AXES

Recall that in previous classes, you have learnt to fix the position of a point in a plane by drawing two mutually perpendicular lines. The fixed point O, where these lines intersect each other is called the **origin** O as shown in Fig. 9.1. These mutually perpendicular lines are called the **coordinate axes**. The horizontal line XOX’ is the **x-axis** or **axis** of x and the vertical line YOY’ is the **y-axis** or **axis** of y.

#### 9.1.1 CARTESIAN COORDINATES OF A POINT

To find the coordinates of a point we proceed as follows. Take X'OX and YOY’ as coordinate axes. Let P be any point in this plane. From point P draw $PA \perp XOX'$ and $PB \perp YOY'$. Then the distance $OA = x$ measured along x-axis and the distance $OB = y$ measured along y-axis determine the position of the point P with reference to these axes. The distance OA measured along the axis of x is called the **abscissa** or x-coordinate and the distance OB (=PA) measured along y-axis is called the **ordinate** or y-coordinate of the point P. The abscissa and the ordinate taken together are called the **coordinates** of the point P. Thus, the coordinates of the point P are (x and y) which represent the position of the point P point in a plane. These two numbers are to form an **ordered pair** because the order in which we write these numbers is important.
In Fig. 9.3 you may note that the position of the ordered pair (3,2) is different from that of (2,3). Thus, we can say that (x,y) and (y,x) are two different ordered pairs representing two different points in a plane.

9.1.2 QUADRANTS

We know that coordinate axes XOX' and YOY' divide the region of the plane into four regions. These regions are called the quadrants as shown in Fig. 9.4. In accordance with the convention of signs, for a point P (x,y) in different quadrants, we have

I quadrant : x > 0, y > 0
II quadrant : x < 0, y > 0
III quadrant : x < 0, y < 0
IV quadrant : x > 0, y < 0

9.2 DISTANCE BETWEEN TWO POINTS

Recall that you have derived the distance formula between two points P (x₁,y₁) and Q (x₂,y₂) in the following manner:

Let us draw a line l || XX’ through P. Let R be the point of intersection of the perpendicular from Q to the line l. Then ΔPQR is a right-angled triangle.

\[ PR = M₁M₂ = OM₂ - OM₁ = x₂ - x₁ \]

and \[ QR = QM₂ - RM₂ \]
Now \( PQ^2 = PR^2 + QR^2 \)  
\[
= (x_2 - x_1)^2 + (y_2 - y_1)^2
\]
\[PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

**Note:** This formula holds for points in all quadrants.

Also the distance of a point \( P(x,y) \) from the origin \( O(0,0) \) is \( OP = \sqrt{x^2 + y^2} \).

Let us illustrate the use of these formulae with some examples.

**Example 9.1** Find the distance between the following pairs of points:

(i) \( A(14,3) \) and \( B(10,6) \)

(ii) \( M(-1,2) \) and \( N(0,-6) \)

**Solution:**

(i) Distance between two points 
\[
= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
Here \( x_1 = 14, y_1 = 3, x_2 = 10, y_2 = 6 \)
\[\therefore \text{Distance between } A \text{ and } B = \sqrt{(-4)^2 + (3)^2}\]
\[= \sqrt{16 + 9}\]
\[= \sqrt{25}\]
\[= 5\]
Distance between \( A \) and \( B \) is 5 units.

(ii) Here \( x_1 = -1, y_1 = 2, x_2 = 0 \) and \( y_2 = -6 \)
Distance between \( A \) and \( B = \sqrt{(0 - (-1))^2 + (-6 - 2)^2}\]
\[= \sqrt{1 + (-8)^2}\]
Example 9.2 Show that the points P(–1, –1), Q(2, 3) and R (–2, 6) are the vertices of a right-angled triangle.

Solution:

\[ PQ^2 = (2 + 1)^2 + (3 + 1)^2 \]
\[ = 3^2 + 4^2 \]
\[ = 9 + 16 = 25 \]

\[ QR^2 = (-4)^2 + (3)^2 \]
\[ = 16 + 9 = 25 \]

and \( RP^2 = 1^2 + (-7)^2 \)
\[ = 1 + 49 \]
\[ = 50 \]

\[ \therefore \quad PQ^2 + QR^2 = 25 + 25 = 50 = RP^2 \]
\[ \implies \Delta PQR \text{ is a right-angled triangle (by converse of Pythagoras Theorem)} \]

Example 9.3 Show that the points A(1, 2), B(4, 5) and C(–1, 0) lie on a straight line.

Solution: Here,

\[ AB = \sqrt{(4 - 1)^2 + (5 - 2)^2} \text{ units} \]
\[ = \sqrt{18} \text{ units} \]
\[ = 3\sqrt{2} \text{ units} \]

\[ BC = \sqrt{(-1 - 4)^2 + (0 - 5)^2} \text{ units} \]
\[ = \sqrt{50} \text{ units} \]
\[ = 5\sqrt{2} \text{ units} \]

and \[ AC = \sqrt{(-1 - 1)^2 + (0 - 2)^2} \text{ units} \]
\[ = \sqrt{4 + 4} \text{ units} \]
\[ = 2\sqrt{2} \text{ units} \]
Now $AB + AC = \left(3\sqrt{2} + 2\sqrt{2}\right)$ units $= 5\sqrt{2}$ units $= BC$

i.e. $BA + AC = BC$

Hence, $A$, $B$, $C$ lie on a straight line. In other words, $A,B,C$ are collinear.

**Example 9.4** Prove that the points $(2a, 4a)$, $(2a, 6a)$ and $\left(2a + \sqrt{3}a, 5a\right)$ are the vertices of an equilateral triangle whose side is $2a$.

**Solution:** Let the points be $A (2a, 4a)$, $B (2a, 6a)$ and $C \left(2a + \sqrt{3}a, 5a\right)$

$AB = \sqrt{(0 + (2a)^2)} = 2a$ units

$BC = \sqrt{(\sqrt{3}a)^2 + (-a)^2} = \sqrt{3a^2 + a^2} = 2a$ units

and $AC = \sqrt{(\sqrt{3}a)^2 + (+a)^2} = 2a$ units

$\Rightarrow$ $AB + BC > AC$, $BC + AC > AB$ and $AB + AC > BC$ and $AB = BC = AC = 2a$

$\Rightarrow$ $A$, $B$, $C$ form the vertices of an equilateral triangle of side $2a$.

**CHECK YOUR PROGRESS 9.1**

1. Find the distance between the following pairs of points.
   (a) $(5, 4)$ and $(2, -3)$
   (b) $(a, -a)$ and $(b, b)$

2. Prove that each of the following sets of points are the vertices of a right angled-triangle.
   (a) $(4, 4)$, $(3, 5)$, $(-1, -1)$
   (b) $(2, 1)$, $(0, 3)$, $(-2, 1)$

3. Show that the following sets of points form the vertices of a triangle:
   (a) $(3, 3)$, $(-3, 3)$ and $(0, 0)$
   (b) $(0, a)$, $(a, b)$ and $(0, 0)$ (if $ab = 0$)

4. Show that the following sets of points are collinear:
   (a) $(3, -6)$, $(2, -4)$ and $(-4, 8)$
   (b) $(0, 3)$, $(0, -4)$ and $(0, 6)$

5. (a) Show that the points $(0, -1)$, $(-2, 3)$, $(6, 7)$ and $(8, 3)$ are the vertices of a rectangle.
   (b) Show that the points $(3, -2)$, $(6, 1)$, $(3, 4)$ and $(0, 1)$ are the vertices of a square.
9.3 SECTION FORMULA

9.3.1 INTERNAL DIVISION

Let \( P(x_1, y_1) \) and \( Q(x_2, y_2) \) be two given points on a line \( l \) and \( R(x, y) \) divide \( PQ \) internally in the ratio \( m_1 : m_2 \).

To find: The coordinates \( x \) and \( y \) of point \( R \).

Construction: Draw \( PL, QN \) and \( RM \) perpendiculars to \( XX' \) from \( P, Q \) and \( R \) respectively and \( L, M \) and \( N \) lie on \( XX' \). Also draw \( RT \bot QN \) and \( PV \bot QN \).

Method: \( R \) divides \( PQ \) internally in the ratio \( m_1 : m_2 \).

\[ \Rightarrow R \text{ lies on } PQ \text{ and } \frac{PR}{RQ} = \frac{m_1}{m_2} \]

Also, in triangles, \( RPS \) and \( QRT \),

\[ \angle RPS = \angle QRT \quad \text{(Corresponding angles as } PS \parallel RT) \] and \[ \angle RSP = \angle QTR = 90^0 \]

\[ \therefore \triangle RPS \sim \triangle QRT \]

\[ \Rightarrow \frac{PR}{RQ} = \frac{RS}{QT} = \frac{PS}{RT} \quad \ldots(i) \]

Also, \( PS = LM = OM - OL = x - x_1 \)

\( RT = MN = ON - OM = x_2 - x \)

\( RS = RM - SM = y - y_1 \)

\( QT = QN - TN = y_2 - y \).

From \((i)\), we have

\[ \therefore \frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y} \]

\[ \Rightarrow m_1(x_2 - x) = m_2(x - x_1) \]

and \( m_1(y_2 - y) = m_2(y - y_1) \)

\[ \Rightarrow x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \]
Thus, the coordinates of \( R \) are:
\[
\left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)
\]

### Coordinates of the mid-point of a line segment

If \( R \) is the mid point of \( PQ \), then,

\[ m_1 = m_2 = 1 \quad \text{(as \( R \) divides \( PQ \) in the ratio 1:1) \]

Coordinates of the mid point are
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

#### 9.3.2 EXTERNAL DIVISION

Let \( R \) divide \( PQ \) externally in the ratio \( m_1:m_2 \)

**To find:** The coordinates of \( R \).

**Construction:** Draw \( PL, QN \) and \( RM \) perpendiculars to \( XX' \) from \( P, Q \) and \( R \) respectively and \( PS \perp RM \) and \( QT \perp RM \).

Clearly, \( \triangle RPS \sim \triangle RQT \).

\[
\frac{RP}{RQ} = \frac{PS}{QT} = \frac{RS}{RT}
\]

or \[
\frac{m_1}{m_2} = \frac{x-x_1}{x-x_2} = \frac{y-y_1}{y-y_2}
\]

\[ \Rightarrow m_1(x-x_2) = m_2(x-x_1) \]

and \[ m_1(y-y_2) = m_2(y-y_1) \]

These give:

\[
x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2} \quad \text{and} \quad y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}
\]

Hence, the coordinates of the point of external division are

\[
\left( \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2} \right)
\]

Let us now take some examples.
Example 9.5 Find the coordinates of the point which divides the line segment joining the points (4, –2) and (–3, 5) internally and externally in the ratio 2:3.

Solution:

(i) Let \( P(x, y) \) be the point of internal division.

\[
\begin{align*}
\therefore x &= \frac{2(-3) + 3(4)}{2 + 3} = \frac{6}{5} \quad \text{and} \quad y = \frac{2(5) + 3(-2)}{2 + 3} = \frac{4}{5} \\
P \text{ has coordinates } &\left( \frac{6}{5}, \frac{4}{5} \right)
\end{align*}
\]

If \( Q(x', y') \) is the point of external division, then

\[
\begin{align*}
x' &= \frac{(2)(-3) - 3(4)}{2 - 3} = 18 \quad \text{and} \quad y' = \frac{(2)(5) - 3(-2)}{2 - 3} = -16 \\
&
\end{align*}
\]

Thus, the coordinates of the point of external division are \((18, -16)\).

Example 9.6 In what ratio does the point \((3, -2)\) divide the line segment joining the points \((1, 4)\) and \((-3, 16)\) ?

Solution: Let the point \( P(3, -2) \) divide the line segment in the ratio \( k : 1 \).

Then the coordinates of \( P \) are \( \left( \frac{-3k + 1}{k + 1}, \frac{16k + 4}{k + 1} \right) \)

But the given coordinates of \( P \) are \((3, -2)\)

\[
\begin{align*}
\therefore \frac{-3k + 1}{k + 1} &= 3 \\
\Rightarrow -3k + 1 &= 3k + 3 \\
\Rightarrow k &= -\frac{1}{3} \\
\Rightarrow P \text{ divides the line segment externally in the ratio } 1:3.
\end{align*}
\]

Example 9.7 The vertices of a quadrilateral \( ABCD \) are respectively \((1, 4)\), \((-2, 1)\), \((0, -1)\) and \((3, 2)\). If \( E, F, G, H \) are respectively the midpoints of \( AB, BC, CD \) and \( DA \), prove that the quadrilateral \( EFGH \) is a parallelogram.

Solution: Since \( E, F, G, \) and \( H \), are the midpoints of the sides \( AB, BC, CD \) and \( DA \), therefore, the coordinates of \( E, F, G, \) and \( H \) respectively are:

\[
\left( \frac{1-2+4+1}{2}, \frac{4+1}{2} \right), \left( \frac{-2+0+1-1}{2}, \frac{1-1-1+2}{2} \right), \left( \frac{0+3-1+2}{2}, \frac{-1+2}{2} \right) \text{ and } \left( \frac{1+3+4+2}{2}, \frac{4+2}{2} \right)
\]
COORDINATE GEOMETRY

Module - II

Cartesian System Of Coordinates

\[ E \left( \frac{-1}{2}, \frac{5}{2} \right), F(-1,0), G \left( \frac{3}{2}, \frac{1}{2} \right) \text{ and } H(2,3) \text{ are the required points.} \]

Also, the mid point of diagonal $EG$ has coordinates

\[ \left( \frac{-1+3}{2}, \frac{5+1}{2} \right) = \left( \frac{1}{2}, \frac{3}{2} \right) \]

Coordinates of midpoint of $FH$ are

\[ \left( \frac{-1+2}{2}, \frac{0+3}{2} \right) = \left( \frac{1}{2}, \frac{3}{2} \right) \]

Since, the midpoints of the diagonals are the same, therefore, the diagonals bisect each other.

Hence $EFGH$ is a parallelogram.

**CHECK YOUR PROGRESS 9.2**

1. Find the midpoint of each of the line segments whose end points are given below:
   (a) $(-2, 3)$ and $(3, 5)$  
   (b) $(6,0)$ and $(-2,10)$

2. Find the coordinates of the point dividing the line segment joining $(-5,-2)$ and $(3,6)$ internally in the ratio 3:1.

3. (a) Three vertices of a parallelogram are $(0,3), (0,6)$ and $(2,9)$. Find the fourth vertex.
   (b) $(4,0), (-4,0), (0,-4)$ and $(0,4)$ are the vertices of a square. Show that the quadrilateral formed by joining the midpoints of the sides is also a square.

4. The line segment joining $(2,3)$ and $(5,-1)$ is trisected. Find the points of trisection.

5. Show that the figure formed by joining the midpoints of the sides of a rectangle is a rhombus.

**9.4 AREA OF A TRIANGLE**

Let us find the area of a triangle whose

vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$

Draw $AL, BM$ and $CN$ perpendiculars to $XX'$.

area of $\triangle ABC$
= Area of trapzium. $BMLA + Area$ of trapzium. $ALNC – Area$ of trapzium. $BMNC$

\[
\frac{1}{2} (BM + AL) ML + \frac{1}{2} (AL + CN) LN - \frac{1}{2} (BM + CN) MN
\]

\[
= \frac{1}{2} (y_2 + y_1)(x_1 - x_2) + \frac{1}{2} (y_1 + y_3)(x_3 - x_1) - \frac{1}{2} (y_2 + y_3)(x_3 - x_2)
\]

\[
= \frac{1}{2} \left[ (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3) \right]
\]

\[
= \frac{1}{2} \left[ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right]
\]

This can be stated in the determinant form as follows:

\[
\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}
\]

**Example 9.8** Find the area of the triangle whose vertices are $A(3, 4), B(6, -2)$ and $C(-4, -5)$.

**Solution:** The area of $\triangle ABC = \frac{1}{2} \begin{vmatrix} 3 & 4 & 1 \\ 6 & -2 & 1 \\ -4 & -5 & 1 \end{vmatrix}$

\[
= \frac{1}{2} [3(-2 + 5) - 4(6 + 4) + 1(-30 - 8)]
\]

\[
= \frac{1}{2} [9 - 40 - 38] = -\frac{69}{2}
\]

As the area is to be positive

\[
\therefore \text{Area of } \triangle ABC = \frac{69}{2} \text{ square units}
\]

**Example 9.9** If the vertices of a triangle are $(1, k), (4, -3)$ and $(-9, 7)$ and its area is 15 square units, find the value(s) of $k$.

**Solution:** Area of triangle $= \frac{1}{2} \begin{vmatrix} 1 & k & 1 \\ 4 & -3 & 1 \\ -9 & 7 & 1 \end{vmatrix}$
Since the area of the triangle is given to be 15,

\[ \frac{-9 - 13k}{2} = 15 \]

or, 

\[-9 - 13k = 30 \]

\[-13k = 39 \]

or, 

\[k = -3\]

**CHECK YOUR PROGRESS 9.3**

1. Find the area of each of the following triangles whose vertices are given below:
   
   (1) (0, 5), (5, -5), and (0,0)
   
   (b) (2, 3), (-2, -3) and (-2, 3)
   
   (c) (a, 0), (0, -a) and (0, 0)

2. The area of a triangle ABC, whose vertices are A (2, -3), B(3, -2) and C \( \left( \frac{5}{2}, k \right) \) is \( \frac{3}{2} \) sq unit. Find the value of k.

3. Find the area of a rectangle whose vertices are (5, 4), (5, -4), (-5, 4) and (-5, -4)

4. Find the area of a quadrilateral whose vertices are (5, -2), (4, -7), (1, 1) and (3, 4)

**9.5 CONDITION FOR COLLINEARITY OF THREE POINTS**

The three points \( A(x_1, y_1), B(x_2, y_2), \) and \( C(x_3, y_3) \) are collinear if and only if the area of the triangle ABC becomes zero.

\[ \frac{1}{2} \left[ x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3 \right] = 0 \]

i.e., 

\[x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3 = 0\]

In short, we can write this result as
Let us illustrate this with the help of examples:

**Example 9.10** Show that the points \(A(a, b + c), B(b, c + a)\) and \(C(c, a + b)\) are collinear.

**Solution**: Area of triangle ABC

\[
\frac{1}{2} \begin{vmatrix} a & b + c & 1 \\ b & c + a & 1 \\ c & a + b & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} \begin{vmatrix} a + b + c & b + c & 1 \\ a + b + c & c + a & 1 \\ a + b + c & a + b & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} (a + b + c) \begin{vmatrix} 1 & b + c & 1 \\ 1 & c + a & 1 \\ 1 & a + b & 1 \end{vmatrix}
= 0
\]

Hence the points are collinear.

**Example 9.11** For what value of \(k\), are the points \((1, 5), (k, 1)\) and \((4, 11)\) collinear?

**Solution**: Area of the triangle formed by the given points is

\[
\frac{1}{2} \begin{vmatrix} 1 & 5 & 1 \\ k & 1 & 1 \\ 4 & 11 & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} \left[ -10 - 5k + 20 + 11k - 4 \right]
\]

\[
= \frac{1}{2} [6k + 6] = 3k + 3
\]

Since the given points are collinear, therefore

\[3k + 3 = 0 \Rightarrow k = -1\]

Hence, for \(k = -1\), the given points are collinear.
**CHECK YOUR PROGRESS 9.4**

1. Show that the points (–1,–1), (5, 7) and (8, 11) are collinear.

2. Show that the points (3, 1), (5, 3) and (6, 4) are collinear.

3. Prove that the points (a, 0), (0, b) and (1, 1) are collinear if \( \frac{1}{a} + \frac{1}{b} = 1 \).

4. If the points \((a, b), (a_1, b_1) \) and \((a – a_1, b – b_1) \) are collinear, show that \( a_1 b = ab \).

5. Find the value of \( k \) for which the points (5, 7), (k, 5) and (0, 2) are collinear.

6. Find the values of \( k \) for which the points \((k, 2k), (–k+1, 2k) \) and \((-4 – k, 6 – 2k) \) are collinear.

**9.6 INCLINATION AND SLOPE OF A LINE**

Look at the Fig. 9.9. The line \( AB \) makes an angle or \( \pi + \alpha \) with the \( x \)-axis (measured in anticlockwise direction).

The *inclination* of the given line is represented by the measure of angle made by the line with the positive direction of \( x \)-axis (measured in anticlockwise direction).

In a special case when the line is parallel to \( x \)-axis or it coincides with the \( x \)-axis, the inclination of the line is defined to be 0°.

![Fig. 9.9](image)

(a) \hspace{2cm} (b) \hspace{2cm} Fig. 9.9

Again look at the pictures of two mountains given below. Here we notice that the mountain in Fig. 9.10 (a) is more steep compared to mountain in Fig. 9.10 (b).
How can we quantify this steepness? Here we say that the angle of inclination of mountain (a) is more than the angle of inclination of mountain (b) with the ground.

Try to see the difference between the ratios of the maximum height from the ground to the base in each case.

Naturally, you will find that the ratio in case (a) is more as compared to the ratio in case (b). That means we are concerned with height and base and their ratio is linked with tangent of an angle, so mathematically this ratio or the tangent of the inclination is termed as slope. We define the slope as tangent of an angle.

The slope of a line is the tangent of the angle θ (say) which the line makes with the positive direction of x-axis. Generally, it is denoted by m (= tan θ)

Note: If a line makes an angle of 90° or 270° with the x-axis, the slope of the line cannot be defined.

Example 9.12 In Fig. 9.9 find the slope of lines AB and BA.

Solution: Slope of line AB = tan α

Slope of line BA = tan (π + α) = tan α.

Note: From this example, we can observe that "slope is independent of the direction of the line segment".

Example 9.13 Find the slope of a line which makes an angle of 30° with the negative direction of x-axis.

Solution:

Here θ = 180° – 30° = 150°
∴ m = slope of the line
    = tan (180° – 30°)
    = - tan 30°
    = \(-\frac{1}{\sqrt{3}}\)
**Example 9.14** Find the slope of a line which makes an angle of $60^0$ with the positive direction of $y$-axis.

**Solution:**

Here $\theta = 90^0 + 60^0$

$\therefore \ m = \text{slope of the line}$

$= \tan (90^0 + 60^0)$

$= -\cot 60^0$

$= -\tan 30^0$

$= -\frac{1}{\sqrt{3}}$

**Example 9.15** If a line is equally inclined to the axes, show that its slope is $\pm 1$.

**Solution:** Let a line $AB$ be equally inclined to the axes and meeting axes at points $A$ and $B$ as shown in the Fig. 9.13

In Fig 9.13(a), inclination of line $AB = \angle XAB = 45^0$

$\therefore$ Slope of the line $AB = \tan 45^0 = 1$

In Fig. 9.13 (b) inclination of line $AB = \angle XAB = 180^0 - 45^0 = 135^0$

$\therefore$ Slope of the line $AB = \tan 135^0 = \tan (180^0 - 45^0) = -\tan 45^0 = -1$

Thus, if a line is equally inclined to the axes, then the slope of the line will be $\pm 1$. 
CHECK YOUR PROGRESS 9.5

1. Find the slope of a line which makes an angle of (i) 60°, (ii) 150° with the positive direction of x-axis.
2. Find the slope of a line which makes an angle of 30° with the positive direction of y-axis.
3. Find the slope of a line which makes an angle of 60° with the negative direction of x-axis.

9.7 SLOPE OF A LINE JOINING TWO DISTINCT POINTS

Let \( A(x_1, y_1) \) and \( B(x_2, y_2) \) be two distinct points. Draw a line through \( A \) and \( B \) and let the inclination of this line be \( \theta \). Let the point of intersection of a horizontal line through \( A \) and a vertical line through \( B \) be \( M \), then the coordinates of \( M \) are as shown in the Fig. 9.14

(A) In Fig. 9.14 (a), angle of inclination \( \angle MAB \) is equal to \( \theta \) (acute). Consequently,

\[
\tan \theta = \tan(\angle MAB) = \frac{MB}{AM} = \frac{y_2 - y_1}{x_2 - x_1}
\]

(B) In Fig. 9.14 (b), angle of inclination \( \theta \) is obtuse, and since \( \theta \) and \( \angle MAB \) are supplementary, consequently,

\[
\tan \theta = -\tan(\angle MAB) = -\frac{MB}{MA} = -\frac{y_2 - y_1}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Hence in both the cases, the slope \( m \) of a line through \( A(x_1, y_1) \) and \( B(x_2, y_2) \) is given by

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]
Note: if \( x_1 = x_2 \), then \( m \) is not defined. In that case the line is parallel to the y-axis.

Is there a line whose slope is 1? Yes, when a line is inclined at 45\(^\circ\) with the positive direction of the x-axis.

Is there a line whose slope is \( \sqrt{3} \)? Yes, when a line is inclined at 60\(^\circ\) with the positive direction of the x-axis.

From the answers to these questions, you must have realised that given any real number \( m \), there will be a line whose slope is \( m \) (because we can always find an angle \( \alpha \) such that \( \tan \alpha = m \)).

**Example 9.16** Find the slope of the line joining the points \( A(6, 3) \) and \( B(4, 10) \).

**Solution**: The slope of the line passing through the points \((x_1, y_1)\) and \((x_2, y_2)\) is \( \frac{y_2-y_1}{x_2-x_1} \).

Here, \( x_1 = 6, \ y_1 = 3; \ x_2 = 4, \ y_2 = 10 \).

Now substituting these values, we have slope \( \frac{10 - 3}{4 - 6} = \frac{-7}{2} \).

**Example 9.17** Determine \( x \), so that the slope of the line passing through the points \( A(3, 6) \) and \( (x, 4) \) is 2.

**Solution**:

\[
\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 6}{x - 3} = \frac{-2}{x - 3}
\]

\[
\therefore \frac{-2}{x - 3} = 2 \quad \text{............. (Given)}
\]

\[
\therefore 2x - 6 = -2 \quad \text{or} \quad x = 2
\]

**CHECK YOUR PROGRESS 9.6**

1. What is the slope of the line joining the points \( A(6, 8) \) and \( B(4, 14) \)?
2. Determine \( x \) so that 4 is the slope of the line through the points \( A(6, 12) \) and \( B(x, 8) \).
3. Determine \( y \), if the slope of the line joining the points \( A(-8, 11) \) and \( B(2, y) \) is \( -\frac{4}{3} \).
4. \( A(2, 3), B(0, 4), \) and \( C(-5, 0) \) are the vertices of a triangle \( ABC \). Find the slope of the line passing through the point \( B \) and the mid point of \( AC \).
5. A(−2, 7), B(1, 0), C(4, 3) and D(1, 2) are the vertices of a quadrilateral ABCD. Show that
   (i) slope of AB = slope of CD  (ii) slope of BC = slope of AD

9.8 CONDITIONS FOR PARALLELISM AND PERPENDICULARITY OF LINES.

9.8.1 Slope of Parallel Lines

Let \( l_1, l_2 \) be two (non-vertical) lines with their slopes \( m_1 \) and \( m_2 \) respectively.

Let \( \theta_1 \) and \( \theta_2 \) be the angles of inclination of these lines respectively.

**Case I**: Let the lines \( l_1 \) and \( l_2 \) be parallel

Then \( \theta_1 = \theta_2 \Rightarrow \tan \theta_1 = \tan \theta_2 \)

\[ \Rightarrow m_1 = m_2 \]

Thus, if two lines are parallel then their slopes are equal.

**Case II**: Let the lines \( l_1 \) and \( l_2 \) have equal slopes.

i.e. \( m_1 = m_2 \Rightarrow \tan \theta_1 = \tan \theta_2 \)

\[ \Rightarrow \theta_1 = \theta_2 \ (0^\circ \leq \theta \leq 180^\circ) \]

\[ \Rightarrow l_1 \parallel l_2 \]

Hence, two (non-vertical) lines are parallel if and only if \( m_1 = m_2 \)

**9.8.2 SLOPES OF PERPENDICULAR LINES**

Let \( l_1 \) and \( l_2 \) be two (non-vertical) lines with their slopes \( m_1 \) and \( m_2 \) respectively. Also let \( \theta_1 \) and \( \theta_2 \) be their inclinations respectively.
Case-I: Let \( l_1 \perp l_2 \)

\[ \Rightarrow \theta_2 = 90^\circ + \theta_1 \quad \text{or} \quad \theta_1 = 90^\circ + \theta_2 \]

\[ \Rightarrow \tan \theta_2 = \tan(90^\circ + \theta_1) \quad \text{or} \quad \tan \theta_1 = \tan(90^\circ + \theta_2) \]

\[ \Rightarrow \tan \theta_2 = -\cot(\theta_1) \quad \text{or} \quad \tan \theta_1 = -\cot(\theta_2) \]

\[ \Rightarrow \tan \theta_2 = -\frac{1}{\tan \theta_1} \quad \text{or} \quad \Rightarrow \tan \theta_1 = -\frac{1}{\tan \theta_2} \]

\[ \Rightarrow \text{In both the cases, we have} \]

\[ \tan \theta_1 \tan \theta_2 = -1 \]

or \( m_1 m_2 = -1 \)

Thus, if two lines are perpendicular then the product of their slopes is equal to \(-1\).

Case II: Let the two lines \( l_1 \) and \( l_2 \) be such that the product of their slopes is \(-1\).

i.e. \( m_1 m_2 = -1 \)

\[ \Rightarrow \tan \theta_1 \tan \theta_2 = -1 \]

\[ \Rightarrow \tan \theta_1 = -\frac{1}{\tan \theta_2} = -\cot \theta_2 = \tan(90^\circ + \theta_2) \]

\[ = \tan(90^\circ + \theta_2). \]

\[ \Rightarrow \text{Either} \quad \theta_1 = 90^\circ + \theta_2 \quad \text{or} \quad \theta_2 = 90^\circ + \theta_1 \]

\[ \Rightarrow \text{In both cases} \quad l_1 \perp l_2. \]

Hence, two (non-vertical) lines are perpendicular if and only if \( m_1 m_2 = -1 \).

Example 9.18 Show that the line passing through the points A(5,6) and B(2,3) is parallel to the line passing, through the points C(9,–2) and D(6,–5).

Solution: Slope of the line AB = \[ \frac{3-6}{2-5} = \frac{-3}{-3} = 1 \]

and slope of the line CD = \[ \frac{-5+2}{6-9} = \frac{-3}{-3} = 1 \]

As the slopes are equal

\[ \therefore \text{AB} \parallel \text{CD}. \]
Example 9.19  Show that the line passing through the points A(2,–5) and B(–2,5) is perpendicular to the line passing through the points L(6,3) and M(1,1).

Solution: Here

\[ m_1 = \text{slope of the line AB} = \frac{5 + 5}{-2 - 2} = \frac{10}{-4} = -\frac{5}{2} \]

and \[ m_2 = \text{slope of the line LM} = \frac{1 - 3}{1 - 6} = \frac{2}{5} \]

Now \[ m_1 \times m_2 = -\frac{5}{2} \times \frac{2}{5} = -1 \]

Hence, the lines are perpendicular to each other.

Example 9.20  Using the concept of slope, show that A(4,4), B(3,5) and C are the vertices of a right triangle.

Solution: Slope of line AB = \[ m_1 = \frac{5 - 4}{3 - 4} = -1 \]

Slope of line BC = \[ m_2 = \frac{-1 - 5}{-1 - 3} = \frac{3}{2} \]

and  slope of line AC = \[ m_3 = \frac{-1 - 4}{-1 - 4} = 1 \]

Now \[ m_1 \times m_3 = -1 \]

⇒ AB ⊥ AC

⇒ ∆ABC is a right-angled triangle.

Hence, A(4,4), B(3,5) and C(−1,−1) are the vertices of right triangle.

Example 9.21  What is the value of \( y \) so that the line passing through the points A(3,\( y \)) and B(2,7) is perpendicular to the line passing through the point \( C(−1,4) \) and D(0,6)?

Solution: Slope of the line AB = \[ m_1 = \frac{7 - y}{2 - 3} = y - 7 \]

Slope of the line CD = \[ m_2 = \frac{6 - 4}{0 + 1} = 2 \]

Since the lines are perpendicular,
\[ \therefore m_1 \times m_2 = -1 \]

or \[ (y - 7) \times 2 = -1 \]

or \[ 2y - 14 = -1 \]

or \[ 2y = 13 \]

or \[ y = \frac{13}{2} \]

Q. **CHECK YOUR PROGRESS 9.7**

1. Show that the line joining the points (2,–3) and (–4,1) is
   (i) parallel to the line joining the points (7,–1) and (0,3).
   (ii) perpendicular to the line joining the points (4,5) and (0,–2).
2. Find the slope of a line parallel to the line joining the points (–4,1) and (2,3).
3. The line joining the points (–5,7) and (0,–2) is perpendicular to the line joining the points (1,3) and (4,x). Find x.
4. A(–2,7), B(1,0), C(4,3) and D(1,2) are the vertices of quadrilateral ABCD. Show that the sides of ABCD are parallel.
5. Using the concept of the slope of a line, show that the points A(6, –1), B(5,0) and C(2,3) are collinear.[Hint: slopes of AB, BC and CA must be equal.]
6. Find \( k \) so that line passing through the points \((k,9)\) and \((2,7)\) is parallel to the line passing through the points \((2,–2)\) and \((6,4)\).
7. Using the concept of slope of a line, show that the points \((-4,–1),\) \((-2–4),\) \((4,0)\) and \((2,3)\) taken in the given order are the vertices of a rectangle.
8. The vertices of a triangle ABC are A(–3,3), B(–1,–4) and C(5,–2). M and N are the midpoints of AB and AC. Show that MN is parallel to BC and \( MN = \frac{1}{2} BC \).

9.9 **INTERCEPTS MADE BY A LINE ON AXES**

If a line \( l \) (not passing through the Origin) meets \( x \)-axis at A and \( y \)-axis at B as shown in Fig. 9.17, then

(i) OA is called the \( x \)-intercept or the intercept made by the line on \( x \)-axis.

(ii) OB is called \( y \)-intercept or the intercept made by the line on \( y \)-axis.
(iii) OA and OB taken together in this order are called the intercepts made by the line $l$ on the axes.

(iv) AB is called the portion of the line intercepted between the axes.

(v) The coordinates of the point A on $x$-axis are $(a,0)$ and those of point B are $(0,b)$

To find the intercept of a line in a given plane on $x$-axis, we put $y = 0$ in the given equation of a line and the value of $x$ so obtained is called the $x$ intercept.

To find the intercept of a line on $y$-axis we put $x = 0$ and the value of $y$ so obtained is called the $y$ intercept.

Note: 1. A line which passes through origin makes no intercepts on axes.

2. A horizontal line has no $x$-intercept and vertical line has no $y$-intercept.

3. The intercepts on $x$-axis and $y$-axis are usually denoted by $a$ and $b$ respectively. But if only $y$-intercept is considered, then it is usually denoted by $c$.

Example 9.22 If a line is represented by $2x + 3y = 6$, find its $x$ and $y$ intercepts.

Solution: The given equation of the line is

$$2x + 3y = 6 \quad \ldots (i)$$

Putting $x = 0$ in $(i)$, we get

$$y = 2$$

Thus, $y$-intercept is 2.

Again putting $y = 0$ in $(i)$, we get

$$2x = 6 \Rightarrow x = 3$$

Thus, $x$-intercept is 3.

CHECK YOUR PROGRESS 9.8

1. Find $x$ and $y$ intercepts, if the equations of lines are:

   $(i) \quad x + 3y = 6 \quad (ii) \quad 7x + 3y = 2 \quad (iii) \quad \frac{x}{2a} + \frac{y}{2b} = 1 \quad (iv) \quad ax + by = c$

   $(v) \quad \frac{y}{2} - 2x = 8 \quad (vi) \quad \frac{y}{3} - \frac{2x}{3} = 7$
9.10 LOCUS OF A POINT

9.10.1 DEFINITION OF THE LOCUS OF A POINT

Locus of a point is the path traced by the point when moving under a given condition or conditions. Thus, locus of a point is a path of definite shape. It may be a straight line, circle or any other curve.

For Example: (i) The locus of a point in a plane which moves such that it is at constant distance from a fixed point in the plane is a circle as shown in Fig. 9.18 (a).

(ii) The locus of a point which moves such that it is always at a constant distance from x-axis is a pair of straight lines parallel to x-axis. [See Fig. 9.18 (b)]

(iii) The locus of a point in a plane which moves such that it is always at a constant distance from the two fixed points in the same plane is perpendicular bisector of the line segment joining the two points. [See Fig. 9.18(c)].

From the definition and the examples of a locus, we observe that

(a) Every point which satisfies the given condition or conditions is a point on the locus.
(b) Every point of the locus must satisfy the given condition or conditions.

9.10.2 EQUATION OF LOCUS

The equation of locus of a moving point \((x,y)\) is an algebraic relation between \(x\) and \(y\) satisfying the given conditions of motion of a point.

The coordinates \((x,y)\) of the moving point which generates the locus are called current coordinates. The point covers all the positions on the locus and is called the general point.

Let us take an example:

Let \(P(4,3)\) and \(Q(7,11)\) be two points. Let us try to locate a point \(R\) which is equidistant from both the points \(P\) and \(Q\).

Let the Coordinates of \(R\) be \((x,y)\).
Then \[ PR = \sqrt{(x-4)^2 + (y-3)^2} = \sqrt{x^2 + y^2 - 8x - 6y + 25} \]

\[ QR = \sqrt{(x-7)^2 + (y-11)^2} = \sqrt{x^2 + y^2 - 14x - 22y + 170} \]

\[ PR = QR \]

\[ \therefore \sqrt{x^2 + y^2 - 8x - 6y + 25} = \sqrt{x^2 + y^2 - 14x - 22y + 170} \]

Squaring, we get
\[ x^2 + y^2 - 8x - 6y + 25 = x^2 + y^2 - 14x - 22y + 170 \]
\[ \Rightarrow 6x + 16y - 145 = 0 \]

This is called the equation of the locus of a point \( R \) which is equidistant from the points \( P \) and \( Q \).

From the above, we observe the following working rule:

9.10.3 WORKING RULE TO FIND THE EQUATION OF THE LOCUS OF A POINT

(i) Take any point \((x, y)\) on the locus.

(ii) Write the given geometrical form in the terms of \( x \) and \( y \) and known constant or constants and simplify it, if necessary.

(iii) Express the given condition in mathematical form in the terms of \( x \) and \( y \) and known constant or constants and simplify it, if necessary.

(iv) The equation so obtained is the equation of the required locus.

**Example 9.23** Find the equation of locus of points which are thrice as far from \((-a, 0)\) as from \((a, 0)\).

**Solution** : Let \( P(x, y) \) be any point on locus. Also let \( A(-a, 0) \) and \( B(a, 0) \) be the two given points.

Then by the given condition.

\[ PA = 3 \ PB \]
\[ \therefore \left( x + a \right)^2 + y^2 = 9 \left( x - a \right)^2 \]
\[ (x + a)^2 + y^2 = a \left( (x - a)^2 + y^2 \right) \]

\[ x^2 + 2ax + a^2 + y^2 = a \left( x^2 - 2ax + a^2 + y^2 \right) \]

\[ 8x^2 - 20ax + 8a^2 + 8y^2 = 0 \]

Thus, \( 2x^2 + 2y^2 - 5ax + 2a^2 = 0 \) is the required equation of the locus.

**Example 9.24** Find the equation of the locus of a point such that the sum of its distances from \((0,2)\) and \((0,-2)\) is 6.

Solution: Let \(P(x, y)\) be any point on the locus. Also let \(A(0,2)\) and \(B(0,-2)\) be the given points.

From the given condition, we have

\[ PA + PB = 6 \]

\[ \therefore \sqrt{x^2 + (y - 2)^2} + \sqrt{x^2 + (y + 2)^2} = 6 \]

or

\[ \sqrt{x^2 + y^2 - 4y + 4} = 6 - \sqrt{x^2 + y^2 + 4y + 4} \]

Squaring both sides, we get

\[ x^2 + y^2 - 4y + 4 = 36 + x^2 + y^2 + 4y + 4 - 12\sqrt{x^2 + y^2 + 4y + 4} \]

or

\[-8y - 36 = -12\sqrt{x^2 + y^2 + 4y + 4} \]

or

\[ 2y + 9 = 3\sqrt{x^2 + y^2 + 4y + 4} \]

Squaring both sides again, we get

\[ (2y + 9)^2 = 9 \left( x^2 + y^2 + 4y + 4 \right) \]
or \[ 4y^2 + 36y + 81 = 9x^2 + 9y^2 + 36y + 36 \]
or \[ 9x^2 + 5y^2 = 45 \]
which is the required equation of the locus.

**Example 9.25** A(3,1) and B(–2,4) are the two vertices of a triangle ABC. Find the equation of the locus of the centroid of the triangle, if the third vertex C is a point of the locus whose equation is \(3x - 4y = 8\).

**Solution** : Let \(C(a,b)\) be the third vertex of the triangle ABC. Since \(C(a,b)\) lies on the locus whose equation is \(3x - 4y = 8\),

\[ \therefore 3a - 4b = 8 \] ... (i)

Let \((h,k)\) be the centroid of the triangle ABC.

\[ \therefore h = \frac{3-2+a}{3} \quad \text{and} \quad k = \frac{1+4+b}{3} \]

\[ \Rightarrow a = 3h - 1 \quad \text{and} \quad b = 3k - 5 \]

Substituting these values of \(a\) and \(b\), we get

\[ 3(3h - 1) - 4(3k - 5) = 8 \]

\[ \Rightarrow 3h - 4k + 3 = 0 \]

Hence, the locus of the point \(G(h,k)\) is \(3x - 4y + 3 = 0\)

**Example 9.26** \((2,-2)\) is a point of the locus whose equation is \(y^2 = ax\). If \((8,b)\) is also a point of locus, find \(b\).

**Solution** : Since \((2,-2)\) is a point of the locus whose equation is \(y^2 = ax\)

\[ \therefore (-2)^2 = 2a \Rightarrow a = 2 \]

\[ \therefore \text{The equation of the locus is } y^2 = 2x. \]

As \((8,b)\) is also a point of this locus

\[ \therefore b^2 = 2 \times 8 \]

\[ \Rightarrow b^2 = 16 \Rightarrow b = \pm 4 \]

Hence, the value of \(b = \pm 2\)
CHECK YOUR PROGRESS 9.9

1. Find the locus of a point which is equidistant from the points (3,4) and (–4,6).
2. Find the locus of a point equidistant from the points (4,2) and the x-axis.
3. Find the equation of locus of a point which moves so that the distance from the point (4,1) is twice its distance from the point (1,5).
4. A (2,3) and B (0,2) are the coordinates of the two vertices of a triangle. Find the locus of a point P such that the area of the triangle PAB = 3 sq. units.
5. Find the equation of the locus of a point which moves so that the sum of the squares of its distances from the point (2,3) and (–3,4) is 16.
6. Find the locus of a point which moves such that the sum of its distances from the points (3,0) and (–3,0) is less than 9.
7. If (h,0) is a point of the locus whose equation is \( x^2 + y^2 - 6x + 8y - 36 = 0 \), find h.
8. If \( \left( \frac{3}{2}, -2 \right) \) is a point of the locus whose equation is \( y^2 = ax \), find b if (b,6) is also a point of the locus.

LET US SUM UP

- Distance between any two points \((x_1, y_1)\) and \((x_2, y_2)\) is \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
- Coordinates of the point dividing the line segment joining the points \((x_1, y_1)\) and \((x_2, y_2)\) internally in the ratio \(m_1 : m_2\) are
  \[
  \left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)
  \]
- Coordinates of the point dividing the line segment joining the points \((x_1, y_1)\) and \((x_2, y_2)\) externally are in the ratio \(m_1 : m_2\) are.
  \[
  \left( \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2} \right)
  \]
- Coordinates of the mid point of the line segment joining the points \((x_1, y_1)\) and \((x_2, y_2)\) are
  \[
  \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
  \]
The area of a triangle with vertices \((x_1, y_1)\) and \((x_2, y_2)\) and \((x_3, y_3)\) is given by

\[
\frac{1}{2} \left[ (x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3) \right]
\]

Three points A, B, and C are collinear if the area of the triangle formed by them is zero.

If \(\theta\) is the angle which a line makes with the positive direction of x-axis, then the slope of the line is \(m = \tan \theta\).

Slope (m) of the line joining \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is given by

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

A line with the slope \(m_1\) is parallel to the line with slope \(m_2\) if \(m_1 = m_2\).

A line with the slope \(m_1\) is perpendicular to the line with slope \(m_2\) if \(m_1 \times m_2 = -1\).

If a line \(l\) (not passing through the origin) meets x-axis at A and y-axis at B then OA is called the x-intercept and OB is called the y-intercept.

Locus of a point is the path traced by it when moving under given condition or conditions.

**SUPPORTIVE WEB SITES**

http://www.wikipedia.org

http://mathworld.wolfram.com

**TERMINAL EXERCISE**

1. Find the distance between the pairs of points:
   (a) \((2, 0)\) and \((1, \cot \theta)\)  
   (b) \((-\sin A, \cos A)\) and \((\sin B, \cos B)\)

2. Which of the following sets of points form a triangle?
   (a) \((3, 2), (-3, 2)\) and \((0, 3)\)  
   (b) \((3, 2), (3, -2)\) and \((3, 0)\)

3. Find the midpoint of the line segment joining the points \((3, -5)\) and \((-6, 8)\).

4. Find the area of the triangle whose vertices are:
   (a) \((1, 2), (-2, 3), (-3, -4)\)  
   (b) \((c, a), (c + a, a), (c - a, -a)\)

5. Show that the following sets of points are collinear (by showing that area formed is 0).
   (a) \((-2, 5), (2, -3)\) and \((0, 1)\)  
   (b) \((a, b + c), (b, c + a)\) and \((c, a + b)\)
6. If \((-3, 12), (7, 6)\) and \((x, a)\) are collinear, find \(x\).

7. Find the area of the quadrilateral whose vertices are \((4,3), (-5,6), (0,7)\) and \((3,-6)\).

8. Find the slope of the line through the points
   (a) \((1,2), (4,2)\)  
   (b) \((4, -6), (-2, -5)\)

9. What is the value of \(y\) so that the line passing through the points \((3, y)\) and \((2,7)\) is parallel to the line passing through the points \((-1, 4)\) and \((0,6)\) ?

10. Without using Pythagoras theorem, show that the points \((4, 4), (3, 5)\) and \((-1, -1)\) are the vertices of a right-angled triangle.

11. Using the concept of slope, determine which of the following sets of points are collinear:
   (i) \((-2, 3), (8, -5)\) and \((5, 4)\),  
   (ii) \((5, 1), (1, -1)\) and \((11, 4)\),

12. If \(A(2, -3)\) and \(B(3, 5)\) are two vertices of a rectangle \(ABCD\), find the slope of
   (i) \(BC\)  
   (ii) \(CD\)  
   (iii) \(DA\).

13. A quadrilateral has vertices at the points \((7, 3), (3, 0), (0, -4)\) and \((4, -1)\). Using slopes, show that the mid-points of the sides of the quadrilateral form a parallelogram.

14. Find the \(x\)-intercepts of the following lines:
   (i) \(2x - 3y = 8\)  
   (ii) \(3x - 7y + 9 = 0\)  
   (iii) \(x - \frac{y}{2} = 3\)

15. Find the equation of the locus of a point equidistant from the points \((2,4)\) and \(y\)-axis.

16. Find the equation of the locus of a point which is equidistant from the points \((a+b, a-b)\) and \((a-b, a+b)\).

17. Is \(A(a,0), B(-a,0)\) are two fixed points, find the locus of a point \(P\) which moves so that \(3|PA| = 2|PB|\).

18. Find the equation of the locus of a point \(P\) if the sum of squares of its distances from \((1,2)\) and \((3,4)\) is 25 units.
ANSWERS

CHECK YOUR PROGRESS 9.1

(a) $\sqrt{58}$ (b) $\sqrt{2(a^2 + b^2)}$

CHECK YOUR PROGRESS 9.2

1. (a) $\left(\frac{1}{2},4\right)$ (b) (2,5) 2. (1,4)

3. (a) (2,6) 4. $\left(\frac{5}{3},\frac{1}{3}\right)$

CHECK YOUR PROGRESS 9.3

1. (a) $\frac{25}{2}$ sq. units (b) 12 sq. units (c) $\frac{a^2}{2}$ sq. units

2. $k = \frac{5}{3}$ 3. 80 sq. units 4. $\frac{41}{2}$ sq. units

CHECK YOUR PROGRESS 9.4

5. $k = 3$ 6. $k = \frac{1}{2}, -1$

CHECK YOUR PROGRESS 9.5

1. (i) $\sqrt{3}$ (ii) $-\frac{1}{\sqrt{3}}$ 2. $-\sqrt{3}$ 3. $-\sqrt{3}$

CHECK YOUR PROGRESS 9.6

1. $-3$ 2. 5 3. $-\frac{7}{3}$ 4. $\frac{5}{3}$

CHECK YOUR PROGRESS 9.7

2. $\frac{1}{3}$ 3. $\frac{14}{3}$ 6. $k = \frac{10}{3}$
CHECK YOUR PROGRESS 9.8

1. $(i)$ $x$-intercept = 6  
   $(ii)$ $x$-intercept = $\frac{2}{7}$  
   $(iii)$ $x$-intercept = $2a$

   $y$-intercept = 2  
   $y$-intercept = $\frac{2}{3}$  
   $y$-intercept = $2b$

2. $(iv)$ $x$-intercept = $\frac{c}{a}$  
   $(v)$ $x$-intercept = $-4$  
   $(vi)$ $x$-intercept = $-\frac{21}{2}$

   $y$-intercept = $\frac{c}{b}$  
   $y$-intercept = 16  
   $y$-intercept = 21

CHECK YOUR PROGRESS 9.9

1. $14x - 4y + 27 = 0$  
2. $x^2 - 8x - 4y + 20 = 0$

3. $3x^2 + 3y^2 - 38y + 87 = 0$  
4. $x - 2y = 2$

5. $x^2 + y^2 + x - 7y + 11 = 0$  
6. $20x^2 + 36y^2 < 405$

7. $3 \pm 3\sqrt{5}$  
8. $\frac{27}{2}$

TERMINAL EXERCISE

1. (a) cosec $\theta$  
   (b) $2 \sin \frac{A + B}{2}$

2. None of the given sets forms a triangle.

3. $\left(-\frac{3}{2}, \frac{3}{2}\right)$  
4. (a) 11 sq. unit  
   (b) $a^2$ sq. unit.

6. $\frac{51 - 5a}{3}$  
7. 29 sq. unit.

8. (a) 0  
   (b) $-\frac{1}{6}$

9. $y = 3$

11. Only $(ii)$

12. $(i)$ $-\frac{1}{8}$  
   $(ii)$ $8$  
   $(iii)$ $-\frac{1}{8}$

14. $(i)$ 4  
   $(ii)$ $-3$  
   $(iii)$ 3

15. $y^2 - 8y - 4x + 20 = 0$

16. $x - y = 0$

17. $5x^2 + 5y^2 - 26ax + 5a^2 = 0$

18. $2x^2 + 2y^2 - 8x - 12y + 5 = 0$